Answer onQuestion #56221, Physics / Mechanics | Relativity

A synchronous satellite circles the earth eastward above equator once every 24h and stays over the same spot on the earth because the earth is rotating at the same rate. What is the orbital radius of the synchronous satellite?

4.2×10^7m

6.4×10^7m

3.6×10^7m

Solution:

Law of Gravitation:

This attractive force is the gravitational force between Earth and the satellite.

Gravity provides the inward pull that keeps the satellite in orbit.

Assuming a circular orbit, the gravitational force must equal the centripetal force.

$$\frac{mv^2}{r} = \frac{Gmm_E}{r^2}$$

where

v = tangential velocity

r = orbit radius = R_E+ h (i.e. not the altitude of the orbit)

R_E= radius of Earth

h = altitude of orbit = height above Earth's surface

m = mass of satellite

m_E= mass of Earth = 5.974*10^24 kg

G = $6.673*10^{-11}$ Nm^2/kg^2

$$v = \sqrt{\left(\frac{Gm_E}{r}\right)}$$

The period of the satellite's orbit is

$$T = \frac{2\pi r}{v} = 2\pi r\sqrt{\frac{r}{Gm_E}} = 2\pi\sqrt{\frac{r^3}{Gm_E}}$$

$$T = 24\ h\ = 24 * 3600\ s$$

Thus,

$$r = \sqrt[3]{Gm_E\left(\frac{T}{2\pi}\right)^2}$$

$$r = \sqrt[3]{(6.673 \cdot 10^{-11}\,\frac{\mathrm{Nm}^2}{\mathrm{kg}^2})(5.974 \cdot 10^{24}\ \mathrm{kg})\left(\frac{24 \cdot 3600\ \mathrm{s}}{2\pi}\right)^2} = 4.22 \cdot 10^7\,\mathrm{m}$$

Answer:4.2×10^7m