Answer on Question #56091-Physics-Other

A cannon of mass 5.75 ✕ 103 kg is rigidly bolted to the earth so it can recoil only by a negligible amount. The cannon fires an 88.0-kg shell horizontally with an initial velocity of +555 m/s. Suppose the cannon is then unbolted from the earth, and no external force hinders its recoil. What would be the velocity of an identical shell fired by this loose cannon? (Hint: In both cases assume that the burning gunpowder imparts the same kinetic energy to the system. Enter your answer to the nearest whole number. Indicate the direction with the sign of your answer.)

Solution

In the first case:

$$K = \frac{mv^2}{2}.$$

In the second case:

$$K = \frac{mv_1^2}{2} + \frac{Mv_2^2}{2}.$$

Since the cannon is free to move we must have conservation of momentum.

$$Mv_2 = mv_1.$$

Then,

$$K = \frac{mv_1^2}{2} + \frac{M}{2}\left(\frac{m}{M}v_1\right)^2 = \frac{mv_1^2}{2} + \frac{m^2v_1^2}{2M} = \frac{mv_1^2}{2}\left(1 + \frac{m}{M}\right).$$

Therefore,

$$\frac{mv^2}{2} = \frac{mv_1^2}{2}\left(1 + \frac{m}{M}\right) \rightarrow v_1 = \frac{v}{\sqrt{1 + \frac{m}{M}}}.$$

$$v_1 = \frac{555}{\sqrt{1 + \frac{88.0}{5.75 \cdot 10^3}}} = 551\frac{m}{s}.$$

Answer: $+551\frac{m}{s}$.