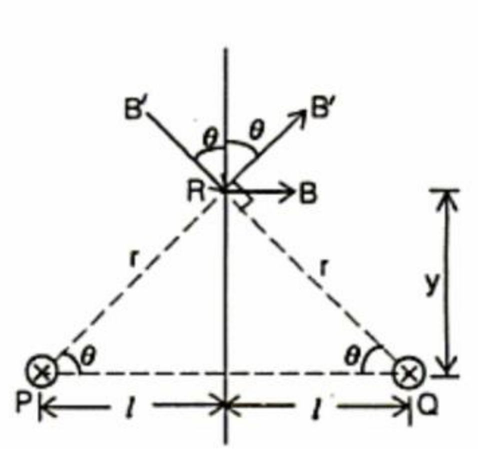
Answer on Question #55908=Physics=Electromagnetism

Each of two long parallel wires carries a constant current i along the same direction. The wires are separated by a distance 2l. The magnitude of resultant magnetic induction in the symmetric plane of this system located between the wires at a distance r from each wire will be? Find the relation between l and r if the magnetic induction in the symmetric plane of the system located between the wires is maximum.

Solution



Situation of the problem is shown in the adjoining figure. Points Q and P represent two wires, each carrying current along inward normal to plane of the paper. It is given that each of these two wires carries a current I and separation between the wires is $2l$. In the figure, dotted line PQ represents the plane of wires and solid line normal to PQ represents the plane of symmetry.

Let magnetic induction in plane of symmetry be maximum at point R, at a distance y from plane of wires P and Q.

Distance of this point from each wire is

$$r = \sqrt{l^2 + y^2}.$$

Magnitude of magnetic induction at R due to each wire is

$$B' = \frac{\mu_0 I}{2\pi r}$$

Direction of these two magnetic inductions at R are as shown in figure. Their components in the plane of symmetry neutralize each other. Therefore, at R, resultant magnetic induction is normal to the plane of symmetry.

The resultant magnetic induction

$$B = 2B' \sin\theta = \frac{\mu_0 I y}{\pi r^2} = \frac{\mu_0 I y}{\pi(l^2 + y^2)} = \frac{\mu_0 I\sqrt{r^2 - l^2}}{\pi r^2} = \frac{\mu_0 I}{\pi r}\sqrt{1 - \frac{l^2}{r^2}}.$$

For B to be a maximum

$$\frac{dB}{dy} = 0 \; or \; y = l.$$

Thus,

$$r^2 = l^2 + l^2 = 2l^2 \rightarrow r = \sqrt{2}l.$$

Answer: $\boldsymbol{\frac{\mu_0 I}{\pi r}\sqrt{1 - \frac{l^2}{r^2}}; r = \sqrt{2}l}$.