Answer on Question #55455, Physics / Optics

A particle moves towards a concave mirror of focal length 30 cm along its axis and with a constant speed of 4 cm/s. What is the speed of its image when the particle is at 90 cm from the mirror?

Solution:

The equation for image formation by rays near the optic axis (paraxial rays) of a mirror has the same form as the thin lens equation:

$$\frac{1}{o} + \frac{1}{i} = \frac{1}{f}$$

where o = object distance, i = image distance, f = focal length.

Differentiation with respect to time

$$\frac{d}{dt}\left(\frac{1}{o}\right) + \frac{d}{dt}\left(\frac{1}{i}\right) = \frac{d}{dt}\left(\frac{1}{f}\right)$$

f=const

Thus,

$$-\frac{1}{o^2}\frac{do}{dt} - \frac{1}{i^2}\frac{di}{dt} = 0$$

$$\frac{di}{dt} = -\frac{i^2}{o^2}\frac{do}{dt}$$

Here:

$$\frac{do}{dt} = 4 \text{ cm/s}$$

$$o = 90 \text{ cm}$$

From mirror equation

$$\frac{1}{i} = \frac{1}{30} - \frac{1}{90} = \frac{2}{90}$$

$$i = \frac{90}{2} = 45 \text{ cm}$$

Hence,

$$\frac{di}{dt} = -\frac{45^2}{90^2} \cdot 4 = -1 \text{ cm/s}$$

Answer: So image is moving at 1 cm/s away from mirror