Answer on Question #55302, Physics Quantum Mechanics

What are the essential conditions for a wave function defining a system must obey and why?

Solution

1. Normalization refers to the fact that every wave function a system must obey the relationship

$$\int_{-\infty}^{+\infty} \Psi(x) \cdot \Psi^*(x)\, dx = \langle \Psi \mid \Psi \rangle = 1 \qquad (1)$$

The origin of this normalization in the Copenhagen interpretation of quantum mechanics. According to this interpretation $|\Psi(r)|^2$ can be interpreted as the probability of finding a particle at a location r. The probability must be normalized to one.

2. For the wave functions of the superposition principle is valid, which consists in the fact that if the system can remain in a state described by the wave functions Ψ_1 and Ψ_2, it can remain in a state described by a wave function $\Psi_\Sigma = C_1\Psi_1 + C_2\Psi_2$ for any complex C_1 and C_2.

Obviously, we can speak of a superposition (addition) of any number of quantum states, that is, the existence of the quantum state of the system, which is described by a wave function $\Psi_\Sigma = \sum_{n=1} c_n \Psi_n$. In this state, the square modulus coefficient c_n determines the probability that the measurement of the system is

detected in a state described by a wave function Ψ_n. Therefore, the normalized wave functions $\sum_{n=1}^{N} c_n c_n^* = 1$.

3. Conditions finiteness of the wave function. The wave function cannot take infinite values, such that the integral ~(1) will be divergent. Therefore, this condition requires that the wave function has square-integrable functions. In particular, problems with normalized wave function of the square of the wave function must tend to zero at infinity.

4. Conditions for the uniqueness of the wave function. The wave function must be single-valued function of the coordinates and time, as the probability density of finding a particle must be determined in each problem is unique. The problems with the use of cylindrical or spherical coordinate system the uniqueness condition leads to the periodicity of the wave functions of the angular variables.