Answer on Question #55269, Physics / Mechanics | Kinematics | Dynamics

A cart moving horizontally along a straight line with constant speed of 30 m/s. A projectile is fired from the moving cart in such a way that it will return to the cart has moved 80 m. At what speed (relative to the cart) and at what angle (to the horizontal) must the projectile be fired?

Solution:

We begin with the equation of motion for the cart, which will be equal to $x = 30t$

Now, we need to note the projection on horizontal axis:

$$v_{projectile-ground} = v_{cart-ground} + v_{cart-projectile}$$

For the projectile the equation of motion will be equal:

$$x = x_0 + v_{projectile-ground} \cdot t$$

$$x = x_0 + v_{cart-ground} \cdot t$$

Thus, the projection on horizontal axis for the cart:

$$v_{projectile-ground} = v_{cart-ground}$$

$$v_{cart-projectile} = 0\ \frac{m}{s}$$

Now, we perform the projection on the vertical axis, we note the same equation relatively for the y axis:

$$v_{projectile-ground} = v_{cart-projectile}$$

$$v_{cart-ground} = 0\ \frac{m}{s}$$

Then, we need to represent the equation of motion in projection on x axis:

First, for the projectile:

$$y = v_{projectile-ground} \cdot t - \frac{gt^2}{2}$$

For the chart:

$$y = 0$$

It is known that a projectile is fired from the moving cart in such a way that it will return to the cart has moved 80 m, so, we can write:

$$30t = 80$$

From the equation noted above, we can determine the time:

$$t = \frac{x}{v_{cart-ground}} = \frac{80m}{30\frac{m}{s}} = 2.667s$$

It should be noted, since the cart is moving at a constant velocity, for a projectile to land back on the cart it would have to be fired at 90 degrees to the horizontal.

Then, we substitute into the formula:

$$y = v_{projectile-ground} \cdot t - \frac{gt^2}{2}$$

$$v_{projectile-ground} \cdot t - \frac{gt^2}{2} = 0$$

Simplify the equation:

$$v_{projectile-ground} \cdot t = \frac{gt^2}{2}$$

Now, we express the $v_{projectile-ground}$:

$$v_{projectile-ground} = \frac{gt}{2} = \frac{9.8\frac{m}{s^2} \cdot 2.667s}{2} = 13.0683\frac{m}{s}$$