Answer on Question #55268, Physics / Mechanics | Kinematics | Dynamics

A racing car of mass 1000kg moves around a banked track at a constant speed of 30ms. Assuming the total reaction at the wheels is normal to the track and the horizontal radius is 100m. Calculate the angle of inclination of the track to the horizontal.

Solution:

First, we should note the given data in accordance with the task. We have: mass of the racing car = 1000 kg, constant speed (v) = 30 m/s, the horizontal radius (r) = 100 m.

We also need to create the graph of the car's motion. The information is provided in Figure 1.

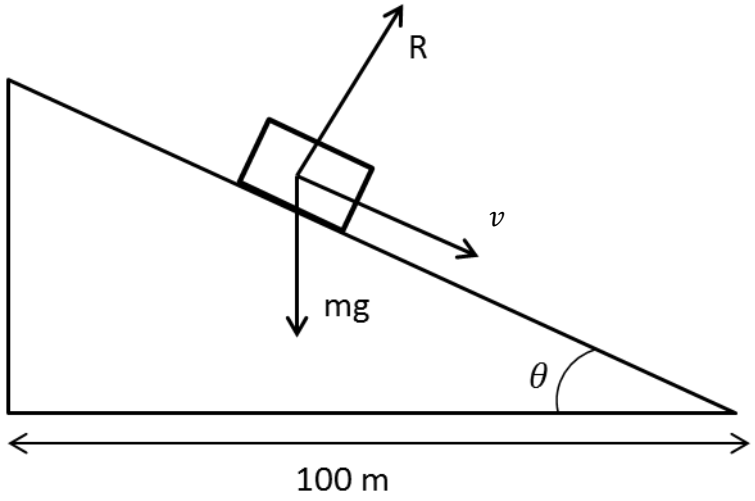

Figure 1 Motion of the car around a banked track.

In accordance with the condition of the task, a racing car of mass m moves around a banked track, the force acting vertically downwards is equal to mg; as can be seen from the Figure 1, R is the reaction force, $\tan\theta = \frac{v^2}{rg}$ (where r is the radius and g is 9.8 $\frac{m}{s^2}$).

Now, we need to construct an appropriate equation, which takes into account all applied forces:

$$\mathrm{R}\cos\theta = \mathrm{mg}$$

We note, horizontally this component would equal the centripetal force:

$$R\cos(90-\theta) = mg$$

$v = rw$, w is the angular speed, r is the radius

We can express w:

$$w = \frac{v}{r}$$

Then, we substitute the angular speed into the formula noted below:

$\mathrm{F} = \mathrm{mrw}^2 = \frac{mrv^2}{r^2} = \frac{mv^2}{r}$, (F is the force towards the centre of the track, r is the radius, w is the angular speed).

Thus, we can rewrite the equation for the forces acting on the car:

$$\mathrm{R}\sin\theta = \frac{\mathrm{mv}^2}{\mathrm{r}}$$

We have noted that $\tan\theta = \frac{v^2}{rg}$, so, we can find an angle:

$$\theta = \tan^{-1}\left(\frac{\mathrm{v}^2}{\mathrm{r}\cdot\mathrm{g}}\right) = \tan^{-1}\left(\frac{\left(30\frac{\mathrm{m}}{\mathrm{s}}\right)^2}{100\mathrm{m}\cdot 9.8\frac{\mathrm{m}}{\mathrm{s}^2}}\right) = \tan^{-1}(0.918) = 42.551°$$

Thus, the angle of inclination of the track to the horizontal will be equal to $42.551°$.