Answer on Question #55265, Physics / Mechanics | Kinematics | Dynamics

A large 3kg object hangs from a rope wound on a 40 kg wheel. The wheel has an actual radius of 0.75m and a radius of gyration of 0.60m. Find (a) the angular acceleration and (b) the distance through which the weight will fall in the first 10s.

Solution:

(a) The net torque is

$$\tau = Fr_1$$

where $r_1 = 0.75$ m and $F = m_1 g = (3\ kg) * (9.8\ m/s^2) = 29.4$ N

Newton's second law for rotation:

$$\tau = I\alpha$$

where I is moment of inertia and α is angular acceleration.

The radius of gyration about a given axis r_g can be computed in terms of the mass moment of inertia I around that axis, and the total mass m;

$$r_g = \sqrt{\frac{I}{m_2}}$$

Thus,

$$I = m_2 r_g^2$$

Hence,

$$\alpha = \frac{\tau}{I} = \frac{m_1 g r_1}{m_2 r_g^2} = \frac{3.0 * 9.8 * 0.75}{40 * 0.60^2} = 1.53\ \text{rad/s}^2$$

(b) The relationship between acceleration of the rope, a, and the angular acceleration, α

$$a = r_1 \alpha$$

The distance through which the weight will fall is

$$h = \frac{at^2}{2} = \frac{r_1 \alpha t^2}{2} = \frac{0.75 * 1.53 * 10^2}{2} \approx 57.4\ \text{m}$$

Answer: (a) $1.53\ \text{rad/s}^2$; (b) 57.4 m