Answer on Question #55229, Physics / Molecular Physics | Thermodynamics

What is the formula to get the steam pressure and steam temperature?

Answer:

A better understanding of the properties of steam may be achieved by understanding the general molecular and atomic structure of matter, and applying this knowledge to ice, water and steam. The molecular arrangement of ice, water, and steam is still not fully understood, but it is convenient to consider the molecules as bonded together by electrical charges (referred to as the hydrogen bond). The degree of excitation of the molecules determines the physical state (or phase) of the substance.

All the three phases of a particular substance can only coexist in equilibrium at a certain temperature and pressure, and this is known as its triple point. The triple point of H_2O, where the three phases of ice, water and steam are in equilibrium, occurs at a temperature of 273.16 K and an absolute pressure of 0.006 112 bar. This pressure is very close to a perfect vacuum. If the pressure is reduced further at this temperature, the ice, instead of melting, sublimates directly into steam.

As the temperature increases and the water approaches its boiling condition, some molecules attain enough kinetic energy to reach velocities that allow them to momentarily escape from the liquid into the space above the surface, before falling back into the liquid.

Further heating causes greater excitation and the number of molecules with enough energy to leave the liquid increases. As the water is heated to its boiling point, bubbles of steam form within it and rise to break through the surface. Considering the molecular arrangement of liquids and vapours, it is logical that the density of steam is much less than that of water, because the steam molecules are further apart from one another. The space immediately above the water surface thus becomes filled with less dense steam molecules.

When the number of molecules leaving the liquid surface is more than those re-entering, the water freely evaporates. At this point it has reached boiling point or its saturation temperature, as it is saturated with heat energy.

If the pressure remains constant, adding more heat does not cause the temperature to rise any further but causes the water to form saturated steam. The temperature of the boiling water and saturated steam within the same system is the same, but the heat energy per unit mass is much greater in the steam.

At atmospheric pressure the saturation temperature is 100°C. However, if the pressure is increased, this will allow the addition of more heat and an increase in temperature without a change of phase.

Therefore, increasing the pressure effectively increases both the enthalpy of water, and the saturation temperature. The relationship between the saturation temperature and the pressure is known as the steam saturation curve. The information is provided in Figure 1.
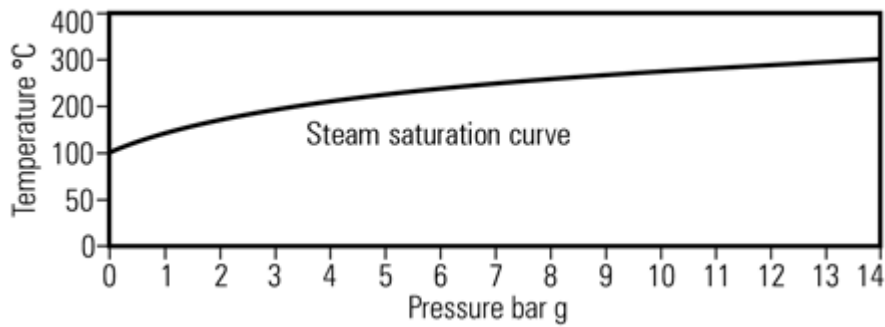
Figure 1 - Steam saturation curve.

Water and steam can coexist at any pressure on this curve, both being at the saturation temperature. Steam at a condition above the saturation curve is known as superheated steam:

- Temperature above saturation temperature is called the degree of superheat of the steam.
- Water at a condition below the curve is called sub-saturated water.

If the steam is able to flow from the boiler at the same rate that it is produced, the addition of further heat simply increases the rate of production. If the steam is restrained from leaving the boiler, and the heat input rate is maintained, the energy flowing into the boiler will be greater than the energy flowing out. This excess energy raises the pressure, in turn allowing the saturation temperature to rise, as the temperature of saturated steam correlates to its pressure.

For ideal gases the relationship between the parameters p, v, and T set by the equation of state:

$$pv = RT$$

Moreover, two of these parameters uniquely define the third. Saturated and superheated vapors in their properties differ significantly from the ideal gas. Therefore, the relation between the parameters p, v, and T and superheated vapor is much more complicated than the equation of state of an ideal gas.

For saturated vapor pressure is a function of the temperature $(P = f(T))$. Thus, for a saturated vapor two variables p and T to determine the state. Moreover, the specific volume v_x is determined by the degree of dryness of the steam's. V_x specific volume is a function of parameters p and x or T and x. Volumes v_b and v_c are functions of temperature and pressure.

Consequently, in order to determine the state of a saturated vapor must be installed depending on the form $p = f(T)$, $v_b = j(p)$, $v_c = f(p)$.

Currently detailed tables compiled for superheated and saturated water vapor to temperatures of 1000°C and pressure of 98 MPa. The tables are a high degree of accuracy. There are three types of tables:

1) the thermodynamic properties of water and steam in the saturated state (temperature);

2) thermodynamic properties of water and steam in a saturation state (pressure);

3) the thermodynamic properties of water and superheated steam.

The first table indicates the temperature of dry saturated steam and boiling water (Celsius and Kelvin) and the corresponding pressure, enthalpy, entropy, heat of vaporization and the specific volume.

The second table indicates the pressure of the dry saturated steam and boiling water, and the corresponding temperature, enthalpy, entropy, heat of vaporization and the specific volume.

The third table for different combinations of temperature and pressure are corresponding enthalpy, entropy and specific volume of water or superheated steam.

We can consider the dependence on the saturated vapor pressure from the temperature. It is assumed that the known: a function of temperature, specific heat of vaporization q and the specific volumes v_1 and v_2, we can find the explicit dependence on the saturated vapor pressure from the temperature.

According to a rough approximation, we can assume that, the value of q does not depend on the temperature and the specific volume of the liquid in comparison with the specific volume of steam can be neglected. In addition, we can assume that the equation is applicable to the vapor state is the Clapeyron equation state $PV = \frac{1}{\mu}RT$. It should also be noted that, these simplifications permissible if the temperature change range is not too wide.

Thus, the following equation is:

$$\frac{dP}{dT} = \frac{q}{Tv} = \frac{\mu q}{RT^2}P,$$

Or

$$\frac{dP}{P} = \frac{\mu q}{R}\frac{dT}{T^2}$$

Integrating, we obtain the following result:

$$lnP = -\frac{\mu q}{RT} + C$$

Constant of the integration C can be found, if the vapor pressure P_0 is known at any one temperature T_0. At this temperature, we get the following:

$$lnP_0 = -\frac{\mu q}{RT_0} + C$$

Excluding the constant C, we obtain:

$$P = P_0 e^{\frac{\mu q}{R}(\frac{1}{T_0}-\frac{1}{T})}$$

For water at atmospheric pressure, the boiling point of T = 373 K, the heat of vaporization $q = 539 \frac{cal}{gK}$; $R = 1.9858 \frac{cal}{mole}$, $\mu = 18$. Substituting into the formula gives us the following:

$$\frac{1}{P}\frac{dP}{dT} = \frac{18 \cdot 539}{1.986 \cdot (373)^2} = 0.0352 K^{-1}$$

It follows that when heated by one degree the pressure of saturated water vapor increases at 0.0352 atm. or 27 mmHg.

Vapor molecules are characterized by a great potential energy. Let b is the work that must be expended against the molecular forces to translate the molecule from an area occupied by the fluid, in the area occupied by the steam.

In accordance with the Boltzmann formula, we obtain the following:

$$n = n_0 e^{-\frac{b}{kT}}$$

Where n is the concentration of vapor molecules, n_0 is the concentration of molecules of the liquid, with it b is independent of temperature. Furthermore, expansion work negligibly $\frac{RT}{\mu}$ compared with the increment of the internal energy.

Then, we obtain the following:

$$n = n_0 e^{-\frac{\mu q}{NkT}} = n_0 e^{-\frac{\mu q}{RT}}$$

Finally, for the saturated steam we obtain:

$$P = nkT = n_0 kT e^{-\frac{\mu q}{RT}}$$

From the noted above formula:

$$lnP = -\frac{\mu q}{RT} + lnT + const$$

The logarithm of T is a slowly varying function of temperature.

If, for example, we consider the moist air, which is located in a closed vessel at 20 degrees and relative humidity f = 80%. And we need to determine, at how much degrees we need to decrease the temperature of the vessel walls, they start to drop the dew. The steam we see as ideal, the specific heat of vaporization of water is equal to = 600 Cal/g, at a temperature of 20 degrees.

In this problem, we replace the derivative of the ratio of finite increments:

$$\frac{T_2 - T_1}{P_2 - P_1} = \frac{RT_1^2}{\mu q P_1}$$

Where P_2 and P_1 are the saturated vapor pressures at temperatures of T_2 and T_1. The vapor pressure in the air at a temperature T_1 and the relative humidity will be f will be fP_1 and therefore, $P_2 = \frac{T_2}{T_1} f P_1$. Substituting these values in the preceding relation, we find:

$$T_2 - T_1 = \frac{f - 1}{\mu q - f R T_1} R T_1^2 = -3.3K$$

To find a more accurate solution, we obtain:

$$ln\frac{fT_2}{T_1} = \frac{\mu q}{RT_1 T_2}(T_2 - T_1)$$

Substituting numerical values and taking the common logarithm, we transform this equation to the form:

$$T_2 - T_1 = 0.124 T_2 \lg(\frac{fT_2}{T_1})$$

To solve the equation, we apply the method of successive approximations. In the zero approximation we believe that $T_2 = T_1$. Using this, we find the first approximation:

$$T_2 - T_1 = 0.124 T_1 \lg f = -3.52K$$

The fourth approach gives us $T_2 - T_1 = -3.67K$. Thus, replacement of the derivative $\frac{dP}{dT}$ by the finite increments results in an error $\sim 10\%$.