Answer on Question #55220, Physics Mechanics Kinematics Dynamics

What is the orbital radius and speed of a synchronous satellite which orbits the earth once every 24 hours. Take G = 6.67 x 10 raise to power -11 Nm/Kg2, mass of the earth is 5.98 x 10 raise to power 24kg.

Solution

Fig.1

Consider a satellite with mass M_S orbiting a central body with a mass of mass M_E. The central body could be a planet, the sun or some other large mass capable of causing sufficient acceleration on a less massive nearby object (in our example body is Earth). If the satellite moves in circular motion, then the net centripetal force acting upon this orbiting satellite is given by the relationship

$$F_{net} = M_S \cdot \frac{v^2}{R} = M_S \cdot \frac{(2\pi R / T)^2}{R} = \frac{4\pi^2 R M_S}{T^2} \quad (1)$$

where R is the orbital radius; v is the speed of satellite

This net centripetal force is the result of the gravitational force that attracts the satellite towards the central body and can be represented as

$$F_{grav} = \frac{G \cdot M_E \cdot M_S}{R^2} \tag{2}$$

Since $F_{grav} = F_{net}$, the above expressions for centripetal force and gravitational force can be set equal to each other. Thus,

$$\frac{4\pi^2 R M_S}{T^2} = \frac{G \cdot M_E \cdot M_S}{R^2} \Rightarrow R = \sqrt[3]{\frac{G \cdot M_E \cdot T^2}{4\pi^2}} \tag{3}$$

Then

$$R = \sqrt[3]{\frac{G \cdot M_E \cdot T^2}{4\pi^2}} = \sqrt[3]{\frac{6.67 \cdot 10^{-11} N\left(m/kg\right)^2 \cdot 5.98 \cdot 10^{24} kg \cdot \left(24 \cdot 3600\right)^2 s^2}{4 \cdot 3.14^2}} = 4.22 \cdot 10^7 m = 42200 km$$

The speed is given by Eq.(4)

$$v = 2\pi R / T = 2 \cdot 3.14 \cdot 42200\,\text{km}/\left(24 \cdot 3600 s\right) = 3.07 km/s \tag{4}$$

$2\pi R$ is the circumference of the orbit.

Answer: $R = \sqrt[3]{\frac{G \cdot M_E \cdot T^2}{4\pi^2}} = 42200 km$; $v = 2\pi R / T = 3.07 km/s$