Answer on Question 55212, Physics, Mechanics | Kinematics | Dynamics

Question:

At what height above the Earth's surface would the acceleration due to gravity be $4.9\,m/s^2$? Assume the mean radius of the Earth is $6.4 \cdot 10^6 m$ and the acceleration due to gravity on the Earth surface is $9.8\,m/s^2$.

Solution:

Let a body of mass m be placed on the Earth surface, whose mass is M and radius is R. Then, the acceleration due to gravity on the Earth surface looks like:

$$g = \frac{GM}{R^2}.$$

Now we placed the body at a height h above the Earth's surface. Then, the acceleration due to gravity above the Earth's surface g' looks like:

$$g' = \frac{GM}{(R+h)^2}.$$

Let's take the ratio between g and g':

$$\frac{g}{g'} = \frac{(R+h)^2}{R^2} = \frac{9.8\,m/s^2}{4.9\,m/s^2} = 2.$$

Then, we get quadratic equation from which we can obtain h:

$$(R+h)^2 = 2R^2,$$

$$R^2 + 2Rh + h^2 = 2R^2,$$

$$h^2 + 2Rh - R^2 = 0.$$

This equation has two roots:

$$D = b^2 - 4ac = (2R)^2 - 4 \cdot 1 \cdot (-R^2) = 4R^2 + 4R^2 = 8R^2,$$

$$h_1 = \frac{-b-\sqrt{D}}{2a} = \frac{-2R-\sqrt{8R^2}}{2} = \frac{-2R-2R\sqrt{2}}{2} = -R - R\sqrt{2}.$$

$$h_2 = \frac{-b+\sqrt{D}}{2a} = \frac{-2R+\sqrt{8R^2}}{2} = \frac{-2R+2R\sqrt{2}}{2} = -R + R\sqrt{2}.$$

Because the height can't be negative the correct answer is $h = -R + R\sqrt{2}$.

Then, we can calculate the height above the Earth's surface at which the acceleration due to gravity would be $4.9\,m/s^2$:

$$h = R\sqrt{2} - R = R(\sqrt{2} - 1) = 0.41R = 0.41 \cdot 6.4 \cdot 10^6 m = 2.624 \cdot 10^6 m.$$

Answer:

$h = 2.624 \cdot 10^6 m.$