Answer on Question55195 - Physics / Mechanics — Kinematics — Dynamics - for completion

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A large $m = 3kg$ object hangs from a rope wound on a $M = 40kg$ wheel. The wheel has an actual radius of $R = 0.75m$ and a radius of gyration of $\rho = 0.60m$. Find the angular acceleration and the distance through which the weight will fall in the first $t_1 = 10s$.

Solution

If a is the acceleration if an object, and T is the tension of the rope, then according to the Newton's second law:

$$ma = mg - T$$

The moment of the force acting on the wheel:

$$M = TR$$

The equation of motion of the wheel:

$$I\varepsilon = M$$

where I is moment of inertia:

$$I = M\rho^2$$

$$a = \varepsilon R$$

From the equations above:

$$M\rho^2\varepsilon = m(g - \varepsilon R)R$$

$$\varepsilon = \frac{mg}{M\rho^2 + mR^2} \approx 1.83 rad/s^2$$

The distance through which the weight will fall in the first 10s:

$$d = \frac{at_1^2}{2} = \frac{\varepsilon R t_1^2}{2} \approx 86.6m$$