

### Question

A ball thrown upwards returns to the ground with the initial speed of  $u$ . The ball is at a height of  $80\text{ m}$  at two times, the time interval being  $6\text{ s}$ . Find  $u$  by assuming  $g = 10\frac{\text{m}}{\text{s}}$ .

### Solution

“Ball returns to the ground with the initial speed”, hence there is no friction.

If there is no friction, we can use the conservation law:  $mgh = \frac{mv^2}{2}$ , where  $m$  – mass of the ball,  $h$  – highest position of the ball,  $v \equiv u$  – initial speed of the ball.

Due to the symmetry of process (once again, because there is no friction) it takes the same time for the ball to move upward from  $80\text{ m}$  to  $h$ , as to move downward from  $h$  to  $80\text{ m}$ .

Thus, we can formulate next equation:

$$h - 80 = \frac{g * 3^2}{2}$$

It comes from general formula (free fall):  $s = \frac{gt^2}{2} + v_0t$ , we consider covered distance ( $s$ ) to be the distance between the highest point and  $80\text{ m}$ , respective time according to the statement above is  $\frac{6}{2} = 3\text{ s}$ , we consider start of movement to be at the reflection point, hence initial speed ( $v_0$ ) is  $0$ .

Plug in  $g$  and solve for  $h$ :

$$h = 80 + \frac{10 * 3^2}{2} = 80 + 5 * 9 = 80 + 45 = 125\text{ (m)}$$

As far as we know  $h$ , we can get  $v$  from the conservation law:

$$mgh = \frac{mv^2}{2}$$

$$v^2 = 2gh$$

$$v = \sqrt{2gh}$$

$$v = \sqrt{2 * 10 * 125} = \sqrt{2 * 2 * 5 * 5^3} = \sqrt{2^2 * 5^4} = 2 * 5^2 = 50\left(\frac{\text{m}}{\text{s}}\right)$$

$$u \equiv v = 50\frac{\text{m}}{\text{s}}$$