If 50 g is suspended at $15^{\circ}$, and 75 g is suspended at $135^{\circ}$, what mass must be suspended at what angle to balance these two forces?

## Solution:



Force $F_{1}=50 \mathrm{~g}$ at $15^{\circ}$ :

$$
\vec{F}_{1}=\left(50 \cos 15^{\circ}, 50 \sin 15^{\circ}\right)=(48.3,12.94)
$$

Force $F_{2}=75 \mathrm{~g}$ at $135^{\circ}$ :

$$
\vec{F}_{1}=\left(75 \cos 135^{\circ}, 75 \sin 135^{\circ}\right)=(-53.03,53.03)
$$

The resultant force is

$$
\begin{gathered}
\vec{R}=\overrightarrow{F_{1}}+\overrightarrow{F_{2}} \\
\vec{R}=(48.3-53.03,12.94+53.03)=(-4.73,65.97)
\end{gathered}
$$

$F_{3}$ is the negative of the resultant $F_{1}$ and $F_{2}$.
So,

$$
\begin{gathered}
\overrightarrow{F_{3}}=-\vec{R} \\
\vec{F}_{3}=(4.73,-65.97)
\end{gathered}
$$

The magnitude of balance force is

$$
F_{3}=\sqrt{4.73^{2}+(-65.97)^{2}}=66.14 \approx 66 \mathrm{~g}
$$

To find direction

$$
\theta=\tan ^{-1}\left(\frac{F_{3 y}}{F_{3 x}}\right)=\tan ^{-1}\left(\frac{-65.97}{4.73}\right)=-85.9^{\circ}=360^{\circ}-85.9^{\circ}=274.1^{\circ}
$$

Answer: 66 g at $274.1^{\circ}$.

