Answer on Question 55062, Physics / Astronomy | Astrophysics

Question:

Derive an expression showing how the of an ultra-relativistic electron emitting synchrotron radiation evolves in time. Assume that 0 is the initial value of and that there is a uniform magnetic field of strength B (and so B _ = B sin a). Your expression should involve only the above variables, time t, and physical constants.

Solution:

The synchrotron power of an electron is given by:

$$P = 2\sigma_T \beta^2 \gamma^2 c \times \frac{B^2}{8\pi} \sin^2 a = A\beta^2\gamma^2 = A(\gamma - 1) \approx A\gamma^2 \text{, where:}$$

$$A = 2\sigma_T c \times \frac{B^2}{8\pi} \text{ is a constant.}$$

Equating this power to the loss in the energy of the electron:

$$\frac{d}{dt}\gamma m_e c^2 = -P$$

$$m_e c^2 \frac{d}{dt}\gamma = -A\gamma^2 \Rightarrow \frac{d\gamma}{\gamma^2} = -\frac{A}{m_e c^2} dt$$

$$\int_{\gamma_0}^{\gamma} \frac{d\gamma}{\gamma^2} = -\frac{A}{m_e c^2} \int_0^t dt$$

$$\left[\frac{1}{\gamma_0} = \frac{1}{\lambda}\right] = -\frac{A}{m_e c^2} t \Rightarrow \gamma = \gamma_0 (1 + A'\gamma_0 t)^{-1}$$

where:

$$A' = \frac{A}{m_e c^2} = \frac{B_\perp \sigma_T}{4\pi m_e c} = \frac{2B_\perp \varepsilon^4}{3m_e^{\ 3} c^5}$$