Question:

Since the optical depth for free-free absorption is proportional to v−2.1 while T is independent of frequency, there must be some frequency above which Thomson scattering in the ISM reduces the observed flux density of an extragalactic point source more than free-free absorption does. Estimate that frequency.

Answer:

$$\left(\frac{EM}{pc \times 10^{-6}}\right) = \int_{los}\left(\frac{N_e}{cm^{-3}}\right)^2 d\left(\frac{s}{pc}\right) = 0.01 \times 1 \times 10^3 = 10$$

We also have:

$$\tau_v \approx 8.235 \times 10^{-2}\left(\frac{T_e}{K}\right)^{-1.35}\left(\frac{v}{GHz}\right)^{-2.1}\left(\frac{EM}{pc \times 10^{-6}}\right) = 6.65 \times 10^{-6}\left(\frac{v}{GHz}\right)^{-2.1}$$

Therefore, the required frequency is given by:

$$\left(\frac{v}{GHz}\right) = \left(\frac{6.65 \times 10^{-6}}{2 \times 10^{-4}}\right)^{\frac{1}{2.1}} = 0.14 \Rightarrow v \approx 140 MHz$$

Answer: v = 140 MHz