Answer on Question#55046 - Physics - Electromagnetism

A uniform charge of $+Q$ is placed on the sphere of a Van de Graaff generator, such that electron discharge is observed from a conducting point a distance of 46 cm away (center to center). Assume the dielectric strength of air, $E = 3 \times 10^6 \frac{\text{N}}{\text{C}}$

(a) Determine the electric potentials at the 100 cm and 80 cm radial location

(b) A small air balloon of charge $q = -8.4\text{nC}$ mass $m = 1.52$ grams, is placed at rest at the 100 cm potential surface, and then released. Determine its speed v at the 80cm radial location (neglect any air resistance and work = ΔK)

<u>Solution:</u>

The electric field at the distance r (greater than its radius) from the center of the charged sphere is given by

$$E = k_e \frac{Q}{r^2},$$

where $k_e = 8.988 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}$ – is the electrostatic constant. Since for $r = 46\text{cm}$ the electric field is given $E = 3 \times 10^6 \frac{\text{N}}{\text{C}}$, we can find the charge of the sphere

$$Q = \frac{E \cdot r^2}{k_e} = \frac{3 \times 10^6 \frac{\text{N}}{\text{C}} \cdot (0.46\text{m})^2}{8.988 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}} = 0.7\mu\text{C}$$

(a) The potential of the charged sphere at the distance r is given by

$$\varphi = k_e \frac{Q}{r}$$

Therefore the potential at $r = 100\text{cm}$ is given by

$$\varphi(100\text{cm}) = 8.988 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2} \frac{0.7\mu\text{C}}{1\text{m}} = 6292\text{V}$$

The potential at $r = 80\text{cm}$ is given by

$$\varphi(80\text{cm}) = 8.988 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2} \frac{0.7\mu\text{C}}{0.8\text{m}} = 7865\text{V}$$

(b) According to the law of conservation of energy, the difference in kinetic energies E_k at these distances is equal to the difference in the potential energies V at these distances of the balloon:

$$E_k(80\text{cm}) - E_k(100\text{cm}) = V(100\text{cm}) - V(80\text{cm})$$

$$E_k(80\text{cm}) - E_k(100\text{cm}) = q \cdot \left(\varphi(100\text{cm}) - \varphi(80\text{cm})\right) =$$

$$= -8.4\text{nC} \cdot (6292\text{V} - 7865\text{V}) = 13.2\mu\text{J}$$

Since $E_k(100\text{cm}) = 0$ and $E_k(80\text{cm}) = \frac{m\cdot v^2}{2}$, we obtain

$$\frac{m \cdot v^2}{2} = 13.2\mu\text{J}$$

$$v = \sqrt{\frac{2 \cdot 13.2\mu\text{J}}{m}} = \sqrt{\frac{26.4\mu\text{J}}{1.52\text{g}}} = 0.13\,\frac{\text{m}}{\text{s}}$$

Answer:

(a) $\varphi(100\text{cm}) = 6292\text{V}$
$\varphi(80\text{cm}) = 7865\text{V}$

(b) $0.13\,\frac{\text{m}}{\text{s}}$