Answer on Question 55023, Physics, Mechanics | Kinematics | Dynamics

Question:

Calculate the mass of the Sun, assuming the Earth's orbit around the Sun is circular, with the radius $r = 1.5 \cdot 10^8 km$.

Solution:

By the universal gravitation equation, the gravitational force acting from the Sun on the Earth looks like:

$$F_{SE} = G\frac{M_{Sun}m_{Earth}}{r^2},$$

where, G is the gravitational constant, M_{Sun} is the mass of the Sun, m_{Earth} is the mass of the Earth and r is the distance between the Sun and the Earth.

Since, we assuming the Earth's orbit around the Sun is circular, the gravitational force F_{SE} must balance the centripetal force on the Earth:

$$F_c = \frac{m_{Earth}v^2}{r},$$

where, m_{Earth} is the mass of the Earth, v is the orbital speed of the Earth and r is the radius of the Earth's orbit.

So, we can write:

$$F_{SE} = F_c,$$

$$G\frac{M_{Sun}m_{Earth}}{r^2} = \frac{m_{Earth}v^2}{r},$$

$$M_{Sun} = v^2\frac{r}{G}.$$

Because the Earth travels around the entire circumference of the circle which is $2\pi r$ in the period T, this means that the orbital speed of the Earth must be $v = \frac{2\pi r}{T}$.

Substituting the expression for the orbital speed into the last equation we can calculate the mass of the Sun:

$$M_{Sun} = \frac{4\pi^2 r^3}{T^2 G} = \frac{4 \cdot 3.14^2 \cdot (1.5 \cdot 10^{11} m)^3}{(3.15 \cdot 10^7 s)^2 \cdot 6.67 \cdot 10^{-11} \frac{Nm^2}{kg^2}} = 2.0 \cdot 10^{30} kg.$$

Answer:

$M_{Sun} = 2.0 \cdot 10^{30} kg.$