Answer on Question #55002-Physics-Other

Ice skaters, ballet dancers, and basketball players executing vertical leaps often give the illusion of "hanging" almost motionless near the top of the leap. To see why this is, consider a leap that takes an athlete up a vertical distance h. Of the total time spent in the air, what fraction is spent in the upper half (i.e., at $y > \frac{1}{2}h$)

Solution

We assume that the height of the leaper (actually, her center of mass) is given by Equation

$$y(t) = y_0 + v_0 t + \frac{at^2}{2},$$

with vertical position y measured positive upward and $a = -g$. Then

$$y(t) - y_0 = v_0 t - \frac{gt^2}{2}.$$

The quadratic formula gives two times when the leaper passes a particular height,

$t_\pm = \frac{1}{g}[v_0 \pm \sqrt{v_0^2 - 2g(y - y_0)}]$, the smaller, t_-, going up and the larger, t_+, going down. The time spent above that height is just $\Delta t(y) = t_+ - t_- = \frac{2}{g}\sqrt{v_0^2 - 2g(y - y_0)}$. The initial velocity for an upward leap of height h is $v_0 = \sqrt{2gh}$, so

$$\Delta t(y) = 2\sqrt{\frac{2}{g}}\sqrt{h - (y - y_0)}.$$

The total time spent in the air is the time spent above the ground, $(y - y_0) = 0$, or $\Delta t(y_0) = 2\sqrt{\frac{2h}{g}}$ and the time spent in the upper half, above $(y - y_0) = \frac{1}{2}h$ is $\sqrt{\frac{1}{2}} = 70.7\%$ of this.