Answer on Question#54967 – Physics – Mechanics | Kinematics | Dynamics

$$(a)\ \sqrt{3} \approx 1.732\left(\frac{m}{s}\right)$$

$$(b)\ \text{atan}(0.15) \approx 8.531°$$

Question

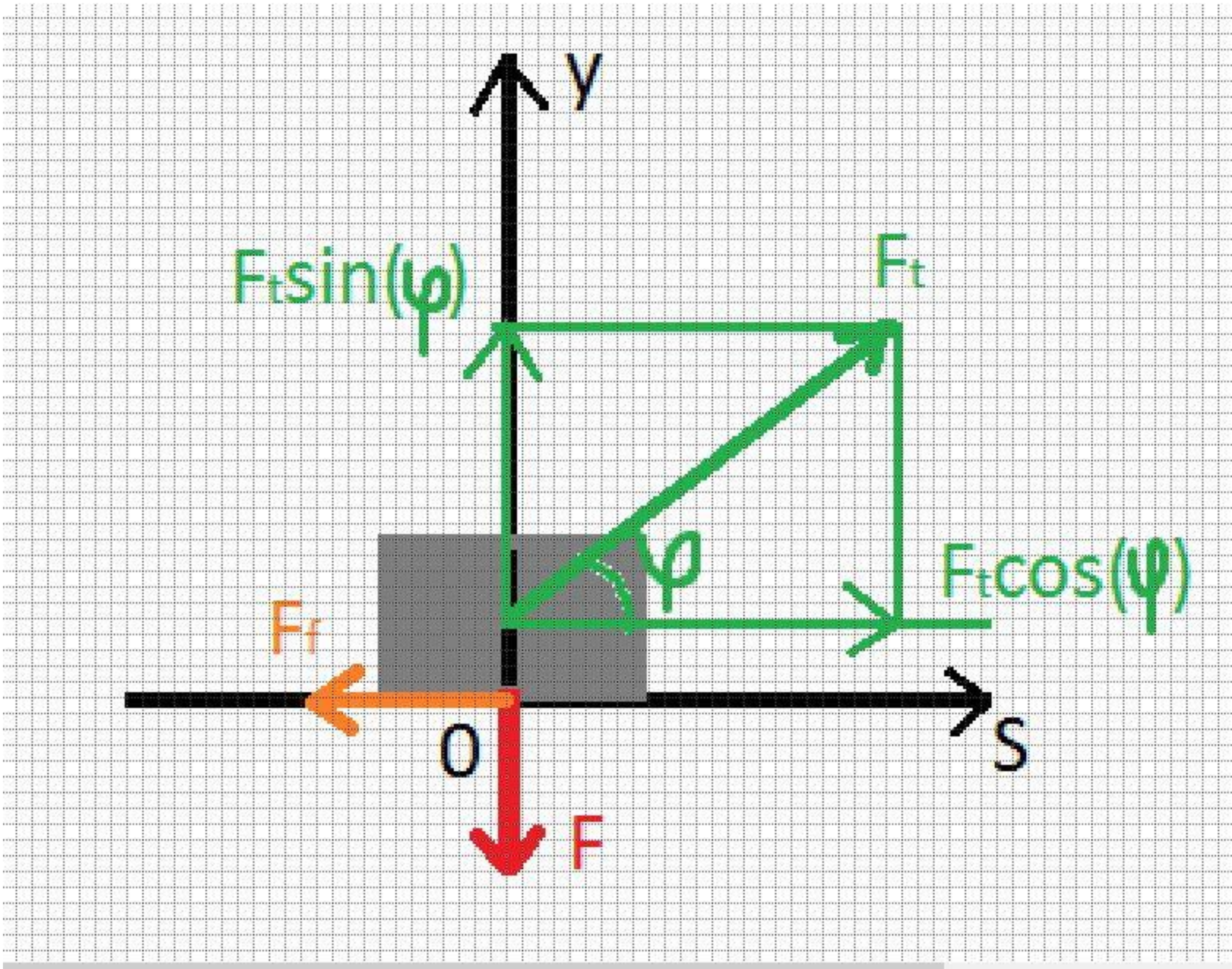
A block of mass $6.00\ kg$ initially at rest is pulled to the right by a constant horizontal force with magnitude $12.0\ N$. The coefficient of kinetic friction between the block and the surface is 0.150.

(a) Find the speed of the block after it has moved $3.0\ m$

(b) Suppose the force F is applied at an angle φ. At what angle should the force applied to achieve the largest possible speed after the block has moved $3.0\ m$ to the right?

Solution

Denote variables: $mass\ (m)$, pulling force (F_t), $friction\ force\ (F_f)$, $gravitation\ force\ (F)$, $gravitation\ acceleration\ (g)$, $acceleration\ of\ the\ block\ (a)$, $coefficient\ of\ kinetic\ friction\ (\mu)$, $velocity\ (V)$, $distance\ (S)$, $angle\ regarding\ surface\ (\varphi)$, $total\ force\ acting\ on\ the\ block\ along\ horizontal\ direction\ (F_{total})$.



(a) Write down relevant formulae:

$$F_f = \mu F$$

$$ma = F_{total}$$

$$F = mg$$

$$F_{total} = F_t - F_f$$

Rearrange expressions:

$$ma = F_t - F_f$$

$$ma = F_t - \mu mg$$

$$a = \frac{F_t}{m} - \mu g$$

Integrate both sides with respect to time:

$$V = \left(\frac{F_t}{m} - \mu g\right) t - C_1$$

At initial moment of time block is not moving:

$$C_1 = 0; \quad V = \left(\frac{F_t}{m} - \mu g\right) t$$

Integrate with respect to time one more time:

$$S = \left(\frac{F_t}{m} - \mu g\right) \frac{t^2}{2} + C_2$$

Assume that at initial moment of time block located at the origin:

$$C_2 = 0; \quad S = \left(\frac{F_t}{m} - \mu g\right) \frac{t^2}{2}$$

Factor out t from expression of S:

$$S^{\frac{1}{2}} = \frac{t\sqrt{2}}{2} \left(\frac{F_t}{m} - \mu g\right)^{\frac{1}{2}}; \quad t = \sqrt{2S} \left(\frac{F_t}{m} - \mu g\right)^{-\frac{1}{2}}$$

Plug it in expression of V:

$$V = \left(\frac{F_t}{m} - \mu g\right) \sqrt{2S} \left(\frac{F_t}{m} - \mu g\right)^{-\frac{1}{2}}$$

$$V = \sqrt{2S} \left(\frac{F_t}{m} - \mu g\right)^{\frac{1}{2}}$$

Substitute in numbers:

$$V = \sqrt{2 * 3} \left(\frac{12}{6} - 0.15 * 10\right)^{\frac{1}{2}} = \sqrt{6}(2 - 1.5)^{\frac{1}{2}} = \frac{\sqrt{6}}{\sqrt{2}} = \sqrt{3} \left(\frac{m}{s}\right)$$

(b) Write down relevant formulae:

$$F_{\perp} = F - F_t \sin(\varphi)$$

$$F_{\parallel} = F_t \cos(\varphi)$$

$$F_f = \mu F_{\perp}$$

$$F = mg$$

$$ma = F_{total}$$

$$F_{total} = F_{\parallel} - F_f$$

Rearrange expressions:

$$ma = F_t \cos(\varphi) - \mu(F - F_t \sin(\varphi))$$

$$ma = F_t \cos(\varphi) - \mu mg + \mu F_t \sin(\varphi)$$

$$a = \frac{F_t \cos(\varphi)}{m} - \mu g + \frac{\mu F_t \sin(\varphi)}{m}$$

Integrate both sides with respect to time:

$$V = \left(\frac{F_t \cos(\varphi)}{m} + \frac{\mu F_t \sin(\varphi)}{m} - \mu g\right) t + C_1$$

At initial moment of time block is not moving:

$$C_1 = 0$$

Introduce new variable $\beta(\varphi)$:

$$\beta = \frac{F_t \cos(\varphi)}{m} + \frac{\mu F_t \sin(\varphi)}{m} - \mu g$$

$$V = \beta t$$

Integrate with respect to time one more time:

$$S = \frac{\beta t^2}{2} + C_2$$

Assume that at initial moment of time block located at the origin:

$$C_2 = 0; \quad S = \frac{\beta t^2}{2}$$

Factor out t from expression of S:

$$t = \sqrt{\frac{2S}{\beta}}$$

Plug it in expression of V:

$$V = \beta\sqrt{\frac{2S}{\beta}} = \sqrt{2S\beta}$$

Find a maximum of V with respect to φ (actually, an extremum, we know that it maximum from physical reasons):

$$\frac{dV}{d\varphi} = \frac{1}{2}\sqrt{\frac{2S}{\beta}}\frac{d\beta}{d\varphi} = \frac{1}{2}\sqrt{\frac{2S}{\beta}}\frac{d\left(\frac{F_t\cos(\varphi)}{m} + \frac{\mu F_t\sin(\varphi)}{m} - \mu g\right)}{d\varphi} =$$

$$\frac{\sqrt{2}}{2}\sqrt{\frac{S}{\beta}}\frac{F_t}{m}(-\sin(\varphi) + \mu\cos(\varphi)) = 0$$

$$\mu\cos(\varphi) = \sin(\varphi)$$

$$\tan(\varphi) = \mu$$

$$\varphi = \operatorname{atan}(\mu)$$

Plug in numbers:

$$\varphi = \operatorname{atan}(0.15)$$