Answer on Question #54941-Physics-Mechanics-Kinematics-Dynamics

The planet Jupiter has an elliptical orbit with e = 0.05 and a semi-major axis of $7.8 \cdot 10^{11}\ m$.

Calculate the energy of the planet, perihelion and aphelion distances and the speed of the planet at these points.

Solution

The energy of the planet is

$$E = \epsilon \frac{mM}{m+M},$$

where ϵ is specific orbital energy, m is mass of Jupiter, M is the mass of Sun.

$$\epsilon = -\frac{G(m+M)}{2a}.$$

$$E = -\frac{G(m+M)}{2a}\frac{mM}{m+M} = -\frac{G(m+M)}{2a} = -\frac{6.67 \cdot 10^{-11}(1.988 \cdot 10^{30} + 1.898 \cdot 10^{27})}{2 \cdot 7.8 \cdot 10^{11}} = -8.5 \cdot 10^{7}\ J.$$

Aphelion distance is

$$r_a = (1+e)a = 8.2 \cdot 10^{11} m.$$

Perihelion distance is

$$r_p = (1-e)a = 7.4 \cdot 10^{11} m.$$

Under standard assumptions the orbital speed v of a body traveling along an elliptic orbit can be computed from the Vis-viva equation as:

$$v = \sqrt{\mu\left(\frac{2}{r} - \frac{1}{a}\right)}.$$

μ is the standard gravitational parameter. For Jupiter

$$\mu = 126686534\frac{km^3}{s^2} = 126686534 \cdot 10^9 \frac{m^3}{s^2}.$$

For aphelion

$$v = \sqrt{126686534 \cdot 10^9\left(\frac{2}{8.2 \cdot 10^{11}} - \frac{1}{7.8 \cdot 10^{11}}\right)} = 383\frac{m}{s}.$$

For perihelion

$$v = \sqrt{126686534 \cdot 10^9\left(\frac{2}{7.4 \cdot 10^{11}} - \frac{1}{7.8 \cdot 10^{11}}\right)} = 424\frac{m}{s}.$$