

Answer on Question #54940-Physics-Mechanics-Kinematics-Dynamics

Three point masses of 3 kg each have the following position vectors:

$$\vec{r}_1 = (2t + 3t^2)m\bar{i} + tm\bar{k}; \vec{r}_2 = 4t^2m\bar{j} + 3m\bar{k}; \vec{r}_3 = (3t - 1)m\bar{i} + 3t^2m\bar{j}$$

Determine the velocity and acceleration of the centre of mass of the system.

Solution

The position of the centre of mass of the system is

$$\bar{x} = \frac{\sum m_i \vec{r}_i}{\sum m_i}.$$

Thus,

$$\bar{x} = \frac{3\vec{r}_1 + 3\vec{r}_2 + 3\vec{r}_3}{3 + 3 + 3} = \frac{\vec{r}_1 + \vec{r}_2 + \vec{r}_3}{3} = \frac{1}{3}(5t - 1 + 3t^2; 7t^2; t + 3)m.$$

The velocity of the centre of mass of the system is

$$\vec{v} = \frac{d\bar{x}}{dt} = \frac{1}{3}(5 + 6t; 14t; 1)\frac{m}{s} = \left(\frac{5 + 6t}{3}; \frac{14t}{3}; \frac{1}{3}\right)\frac{m}{s}.$$

The acceleration of the centre of mass of the system is

$$\vec{a} = \frac{d\vec{v}}{dt} = \left(2; \frac{14}{3}; 0\right)\frac{m}{s}.$$