Answer on Question #54840-Physics-Other

For surface waves in shallow water, the frequency and wavelength are connected through the relation

$$f = \sqrt{\frac{2\pi S}{\rho \lambda^3}}$$

where S and ρ respectively denote surface tension and density of water. Calculate the group velocity of waves. How is it related to phase velocity?

Solution

$$v_g = \frac{d\omega}{dk}.$$

$$\omega = 2\pi f, k = \frac{2\pi}{\lambda} \rightarrow \lambda = \frac{2\pi}{k}$$

$$v_g = \frac{d}{dk}\left(\sqrt{\frac{Sk^3}{\rho}}\right) = \frac{3}{2}\sqrt{\frac{Sk}{\rho}} = \frac{3}{2}\sqrt{\frac{2\pi S}{\rho \lambda}} = \frac{3}{2}\lambda f.$$

The phase velocity is

$$v_p = \lambda f.$$

Thus,

$$v_g = \frac{3}{2}v_p.$$