Answer on Question #54837, Physics Mechanics Kinematics Dynamics

The potential energy (in J) of a system in one dimension is given by: 2 3 U (x) = 5 − x + 3x − 2x

What is the work done in moving a particle in this potential from x = 1 m to x = 2 m? What is the force on a particle in this potential at x = 1 m and x = 2 m? Locate the points of stable and unstable equilibrium for this system.

Solution:

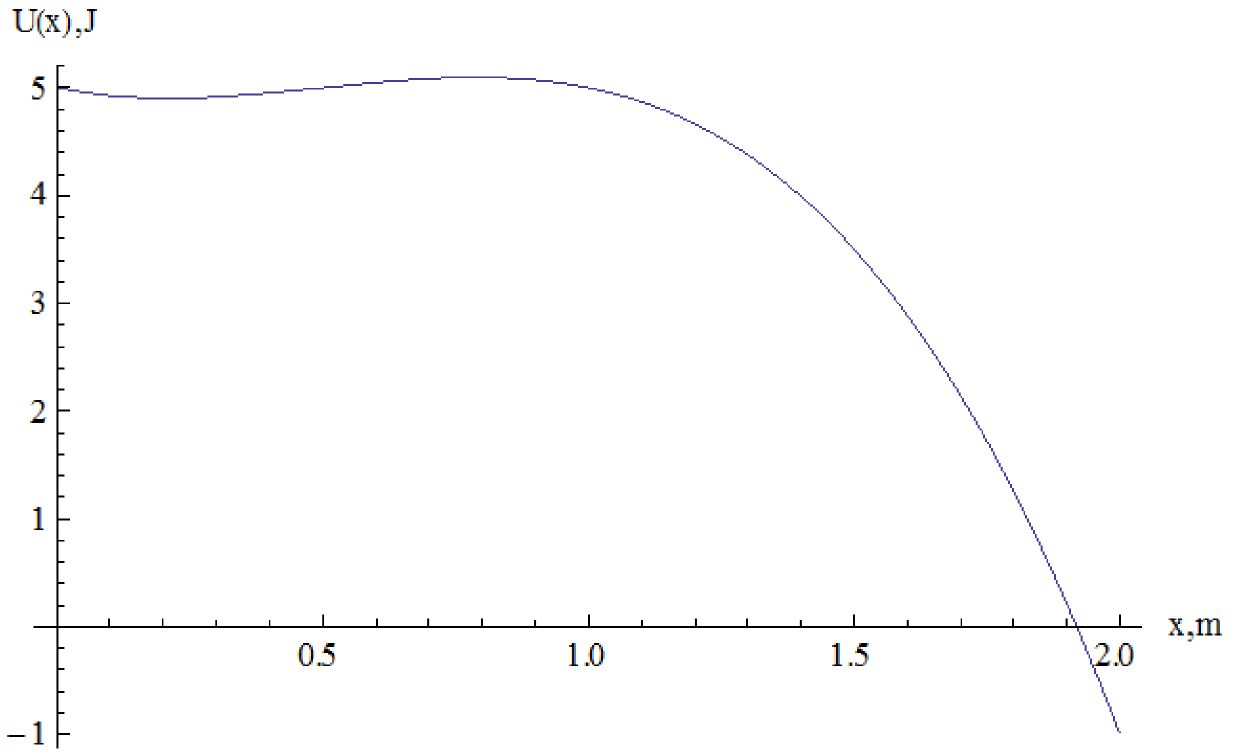

Fig.1

The work done in moving a particle in this potential from x = 1 m to x = 2 m is given by Eq. (1)

$$A = -\left(U(x=2) - U(x=1)\right) = -\left[\left(5 - 2 + 3\cdot 2^2 - 2\cdot 2^3\right) - \left(5 - 1 + 3\cdot 1^2 - 2\cdot 1^3\right)\right] = 6A \quad (1)$$

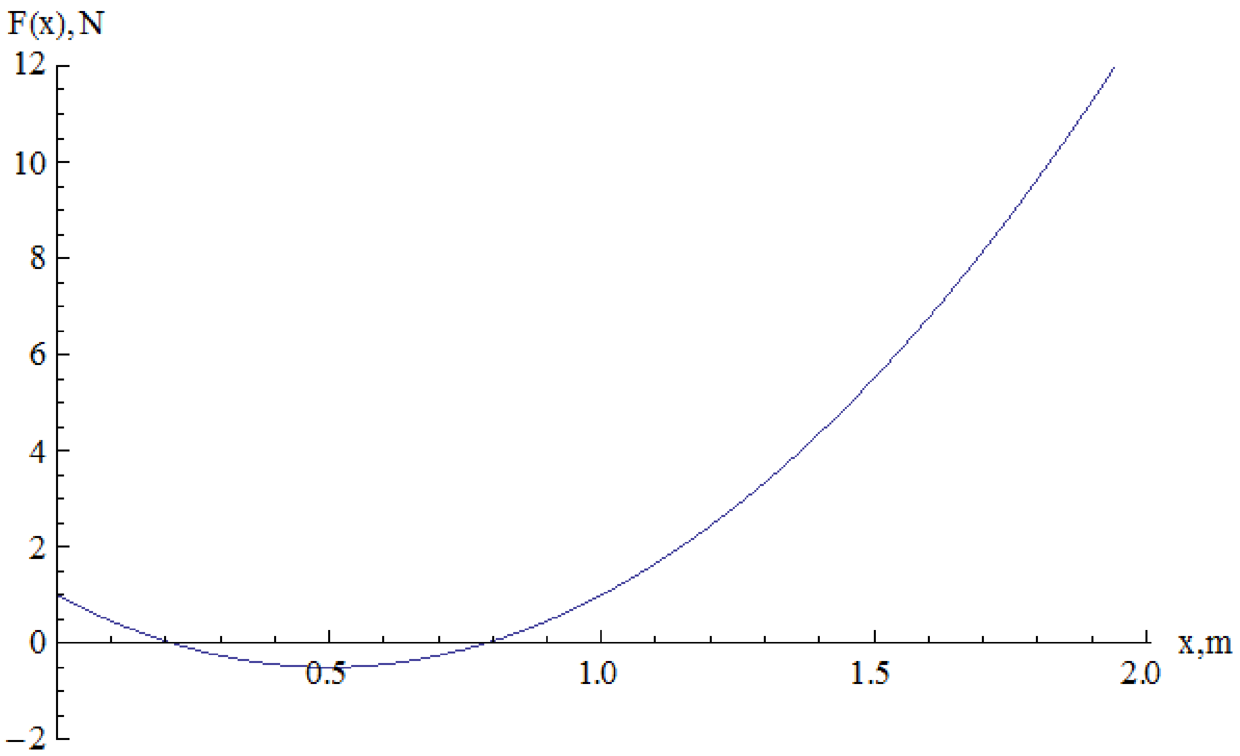

Fig.2

The force on a particle in this potential

$$F(x) = -\frac{\partial U(x)}{\partial x} = -\frac{\partial}{\partial x}\left(5 - x + 3x^2 - 2x^3\right) = -\left(-1 + 6x - 6x^2\right) = 1 - 6x + 6x^2 \qquad (2)$$

The force at x = 1 m and x = 2 m

$$\begin{aligned} F(1) &= 1 - 6\cdot 1 + 6\cdot 1^2 = 1N \\ F(2) &= 1 - 6\cdot 2 + 6\cdot 2^2 = 13N \end{aligned} \qquad (3)$$

The points of stable and unstable equilibrium for this system

$$\frac{\partial U(x)}{\partial x} = 0 \Rightarrow \frac{\partial}{\partial x}\left(5 - x + 3x^2 - 2x^3\right) = \left(-1 + 6x - 6x^2\right) = -1 + 6x - 6x^2 = 0$$

$$6x^2 - 6x + 1 = 0 \Rightarrow D = 6^2 - 4 \cdot 6 \cdot 1 = 6$$

$$x_1 = \frac{6+\sqrt{6}}{12} = 0.5 + \frac{\sqrt{6}}{12} \approx 0.704$$

$$x_2 = \frac{6-\sqrt{6}}{12} = 0.5 - \frac{\sqrt{6}}{12} \approx 0.296$$

$$\frac{\partial^2 U(x)}{\partial x^2} = 6 - 12x$$

$$\frac{\partial^2 U(x_1)}{\partial x^2} = -2.448 \Rightarrow \text{max}\ \ (\text{unstable point } x_1 = \frac{6+\sqrt{6}}{12} \approx 0.704)$$

$$\frac{\partial^2 U(x_2)}{\partial x^2} = +2.448 \Rightarrow \text{min}\ \ (\text{stable point } x_2 = \frac{6-\sqrt{6}}{12} \approx 0.296)$$