Answer on Question #54836-Physics-Mechanics-Kinematics-Dynamics

Three point masses of 3 kg each have the following position vectors:

$$\bar{r_1} = (2t + 3t^2)m\bar{\imath} + tm\bar{k}; \bar{r_2} = 4t^2m\bar{\jmath} + 3m\bar{k}; \bar{r_3} = (3t - 1)m\bar{\imath} + 3t^2m\bar{\jmath}$$

Determine the velocity and acceleration of the centre of mass of the system.

Solution

The position of the centre of mass of the system is

$$\bar{x} = \frac{\sum m_i \bar{r_i}}{\sum m_i}.$$

Thus,

$$\bar{x} = \frac{3\bar{r_1} + 3\bar{r_2} + 3\bar{r_3}}{3 + 3 + 3} = \frac{\bar{r_1} + \bar{r_2} + \bar{r_3}}{3} = \frac{1}{3}(5t - 1 + 3t^2; 7t^2; t + 3)m.$$

The velocity of the centre of mass of the system is

$$\bar{v} = \frac{d\bar{x}}{dt} = \frac{1}{3}(5 + 6t; 14t; 1)\frac{m}{s} = \left(\frac{5 + 6t}{3}; \frac{14t}{3}; \frac{1}{3}\right)\frac{m}{s}.$$

The acceleration of the centre of mass of the system is

$$\bar{a} = \frac{d\bar{v}}{dt} = \left(2; \frac{14}{3}; 0\right)\frac{m}{s}.$$