Answer on Question #54781, Physics / Mechanics | Kinematics | Dynamics

A cart is moving horizontally along a straight line with constant speed of 30 m/s. A projectile is fired from the moving cart in such a way that it will return to the cart after the cart has moved 80 m. At what speed (relative to the cart) and at what angle (to the horizontal) must the projectile be fired?

Solution:

Equation of motion in projection on OX:

For projectile:

$$x = x_0 + v_{projectile-groundX} * t$$

For cart:

$$x = x_0 + v_{cart-groundX} * t$$

$$v_{projectile-groundX} = v_{cart-groundX}$$

Thus, the relative velocity of projectile on x-direction (relative to the cart) is

$$v_{cart-projectileX} = 0 \text{ m/s}$$

The time of cart moving

$$t = \frac{d}{v} = \frac{80\ m}{30\ m/s} = 2.667 \text{ s}$$

In projectile motion, the horizontal motion and the vertical motion are independent of each other; that is, neither motion affects the other.

The horizontal component of the velocity of the object remains unchanged throughout the motion. The vertical component of the velocity increases linearly, because the acceleration due to gravity is constant (g=9.81 m/s^2).

The equation of motion in OY projection:

$$y = y_0 + v_{0y}t - \frac{1}{2}gt^2$$

In our case $y = y_0 = 0$

Thus,

$$0 = v_{0y}t - \frac{1}{2}gt^2$$

$$v_{0y} = \frac{1}{2}gt = \frac{1}{2} * 9.81 * 2.667 = 13.082 \text{ m/s}$$

The initial velocity is

$$v_0 = v_{0y}^2 \approx 13.1 \text{ m/s}$$

The angle is

$$\theta = \tan^{-1}\left(\frac{v_{0y}}{v_{0x}}\right) = \tan^{-1}\left(\frac{13.082}{0}\right) = \tan^{-1}(\infty) = 90°$$

Answer: 13.1 m/s at 90° to horizontal.