Answer on Question #54771, Physics Mechanics Kinematics Dynamics

What are the conditions for the mechanical equilibrium of a rigid body?

A ladder is placed against a wall making an angle θ with the floor. The wall is frictionless but the coefficients of static and kinetic friction for the floor are μs and μk, respectively.

Obtain an expression for the smallest value of θ for the ladder to not slip.

Answer:

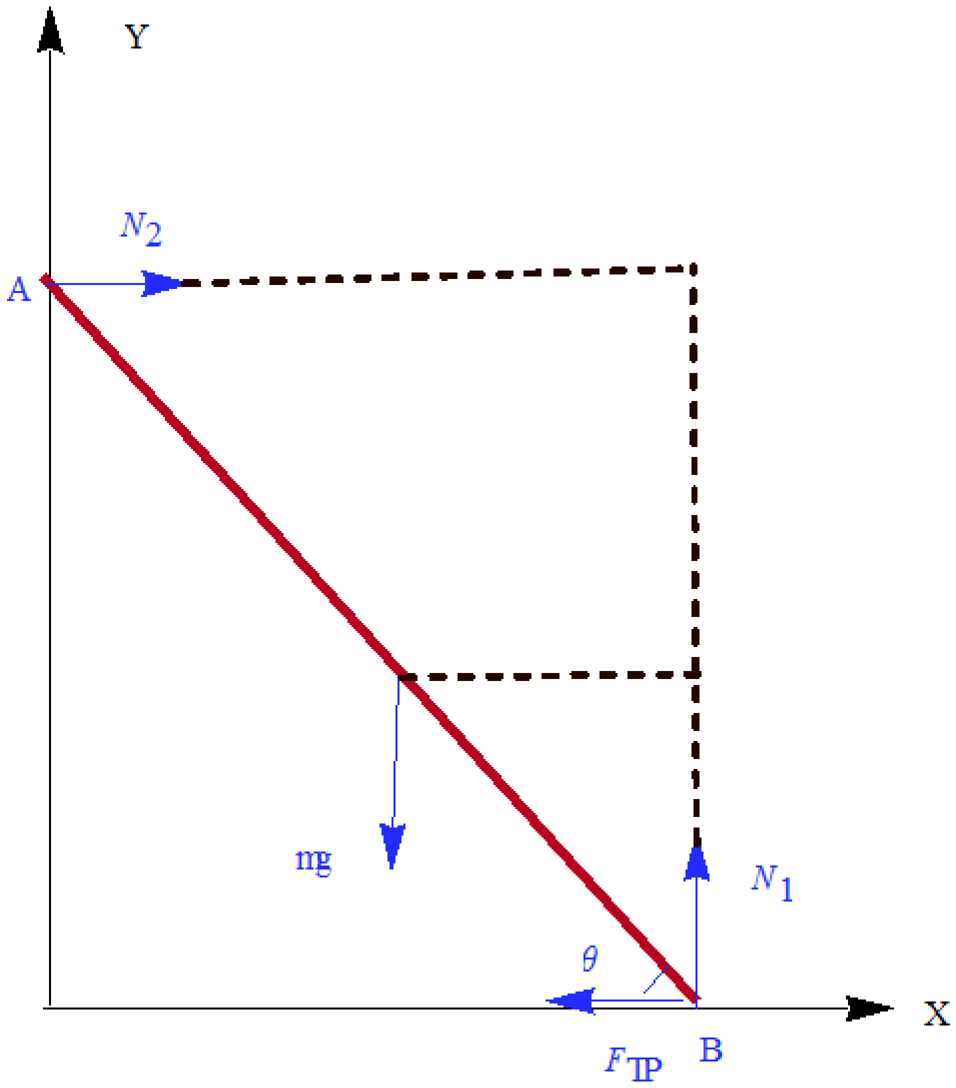

Fig.1

According to the second Newton's law (see Fig.1)

$$m\vec{g} + \vec{N}_1 + \vec{N}_2 + \vec{F}_{TP} = 0 \qquad (1)$$

where m is the mass of ladder; $g = 9.8 m/s^2$ is the gravity acceleration; $\vec{N}_1$ is the reaction force of floor; $\vec{N}_2$ is the reaction force of wall; $F_{TP} = N_1\mu_S$ is the force of friction between ladder and floor.

The projections on the axis

$$\begin{cases} OX: & -mg + N_1 = 0 \\ OY: & N_2 - F_{TP} = 0 \end{cases} \quad (2)$$

So,

$$\begin{cases} N_1 = mg \\ N_2 = N_1\mu_S = mg\mu_S \end{cases} \quad (3)$$

Equality torques relative to point B:

$$\sum_i \vec{M}_{Ai} = 0 \Rightarrow mg\cdot\frac{L}{2}\cdot\cos\theta - N_2\cdot L\sin\theta = 0 \quad (4)$$

where L is the length of the ladder; θ is the angle between the ladder and the floor.

From (3) and (4):

$$\begin{cases} N_2 = mg\mu_S \\ mg\cdot\frac{L}{2}\cdot\cos\theta = N_2\cdot L\sin\theta \end{cases} \Rightarrow mg\cdot\frac{L}{2}\cdot\cos\theta = mg\mu_S\cdot L\sin\theta \Rightarrow$$

$$\cos\theta/\sin\theta = 2\mu_S \Rightarrow \cot\theta = 2\mu_S \Rightarrow \theta = \text{arccot}(2\mu_S)$$

$$\theta_{\min} = \text{arccot}(2\mu_S) \quad (5)$$

Answer: $\theta_{\min} = \text{arccot}(2\mu_S)$