

Answer on Question #54595-Physics- Electromagnetism

Do the following fields satisfy all four Maxwell's equations?

$$\vec{E}(t) = \vec{E}_0 \sin x \sin t, \vec{B} = \vec{B}_0 \cos x \cos t.$$

Solution

$$1) \nabla \cdot \vec{B} = \frac{\partial}{\partial x} (B_{0x} \cos x \cos t) = B_{0x} \cos t \frac{\partial}{\partial x} (\cos x) = -B_{0x} \sin x \cos t = 0 \forall t \text{ only if } B_{0x} = 0.$$

$$2) \nabla \times \vec{E}(t) = \vec{k} \frac{\partial}{\partial x} (E_{0y} \sin x \sin t) - \vec{j} \frac{\partial}{\partial x} (E_{0z} \sin x \sin t) = (0; -E_{0z} \cos x \sin t; E_{0y} \cos x \sin t).$$

$$\frac{\partial \vec{B}}{\partial t} = -\vec{B}_0 \cos x \sin t.$$

Thus

$$\nabla \times \vec{E}(t) = -\frac{\partial \vec{B}}{\partial t} \frac{1}{c} \text{ only if } B_{0y} = -cE_{0z} \text{ and } B_{0z} = cE_{0y}.$$

$$3) \nabla \cdot \vec{E} = \frac{\partial}{\partial x} (E_{0x} \sin x \sin t) = E_{0x} \cos t \frac{\partial}{\partial x} (\sin x) = E_{0x} \cos x \cos t = 0 \forall t \text{ only if } E_{0x} = 0.$$

$$4) \frac{\partial \vec{E}}{\partial t} = \vec{E}_0 \sin x \cos t.$$

$$\nabla \times \vec{B}(t) = \vec{k} \frac{\partial}{\partial x} (B_{0y} \cos x \cos t) - \vec{j} \frac{\partial}{\partial x} (B_{0z} \cos x \cos t) = (0; B_{0z} \sin x \cos t; -B_{0y} \sin x \cos t).$$

So,

$$\nabla \times \vec{B}(t) = \frac{1}{c} \frac{\partial \vec{E}}{\partial t} \text{ only if } cB_{0y} = -E_{0z} \text{ and } cB_{0z} = E_{0y}.$$

So, $cB_{0y} = \frac{1}{c} B_{0y} \rightarrow c = \frac{1}{c}$. This is not true. Thus $\vec{E}_0 = \vec{B}_0 = \vec{0}$.

Therefore the following fields don't satisfy all four Maxwell's equations.

(If we work in system where $c = \frac{1}{c} = 1$ the following fields satisfy all four Maxwell's equations only if

$\vec{E}_0 = (0; E; -B)$ and $\vec{B}_0 = (0; B; E)$, where E and B are arbitrary constants.)