Answer on Question #54539, Physics / Electromagnetism

A uniform plane wave travels through free space, and its instantaneous electric field intensity vector is expressed as

E=15cos(ωt+10πz+θ) y^ V/m

Where time is in seconds, position z is in meters. The magnetic field intensity of the wave is H=0.02 A/m at t = 0 and z = 1.15 m. Determine

(a) The operating frequency and initial (for t = 0) phase θ of the electric field in the plane z = 0.
(b) Expression for the instantaneous magnetic field intensity vector
(c) The complex magnetic field intensity vector

Solution:
(a)
As the propagation medium is free space and the phase coefficient of the wave is
β = 10π rad/m, the operating frequency is

$$f = \frac{\omega}{2\pi} = \frac{\beta c_0}{2\pi} = \frac{10\pi c_0}{2\pi} = 5c_0 = 5 * 3 * 10^8 = 1.5 * 10^9 \text{ Hz} = 1.5 \text{ GHz}$$

The magnetic field intensity of the wave amounting to H = 0.02 A/m at t = 0 and z = 1.15 m, the initial (for t = 0) phase in the plane z = 0 of the electric field intensity (θ_0) turns out to be

$$H_x(z,t) = -\sqrt{\frac{\varepsilon_0}{\mu_0}} E_y(z,t)$$

$$H = 0.02\ \frac{A}{m} = -\frac{15}{377}\cos(10\pi * 1.15 + \theta_0)$$

$$\cos(11.5\pi + \theta_0) = -0.503$$

$$11.5\pi + \theta_0 = \cos^{-1}(-0.503)$$

$$11.5\pi + \theta_0 = -\frac{\pi}{3}$$

$$\theta_0 = -11.5\pi - \frac{\pi}{3} = \frac{\pi}{2} - \frac{\pi}{3} - 12\pi$$

$$\theta_0 = \frac{\pi}{2} - \frac{\pi}{3} = \frac{\pi}{6} = 30°$$

(b)

$$H_x(z,t) = -\sqrt{\frac{\varepsilon_0}{\mu_0}} E_y(z,t)$$

$$\omega = 2\pi * 1.5 * 10^9 = 3\pi * 10^9 \text{ rad/s}$$

$$H = -\frac{15}{377}\cos(3\pi * 10^9 t + 10\pi z + 30°)\hat{\mathbf{x}} \text{ A/m}$$

(c)

We first go to exponential notation,

$$H = Re\left\{-\frac{15}{377}e^{j\left(3\pi*10^9 t+10\pi z+30°\right)}\hat{\mathbf{x}}\right\} \text{ A/m}$$

$$H = -\frac{15}{377}e^{j(10\pi z+30°)}\hat{\mathbf{x}} \text{ A/m}$$