Answer on Question #54322-Physics-Electrodynamics

Consider the energy and fluctuation in energy of an arbitrary system in contact with a heat reservoir at absolute temperature $T = \frac{1}{k\beta}$. Calculate dispersion of energy.

Solution

The average energy of the system is

$$\bar{E} = -\frac{\partial \ln z}{\partial \beta}$$

where $z = \sum_n \exp(-\beta E_n)$ sums over all states of the system.

$$\overline{E^2} = \frac{\sum_n \mathrm{E_n^2} \exp(-\beta E_n)}{\sum_n \exp(-\beta E_n)} = \frac{\frac{\partial^2 z}{\partial \beta^2}}{z} = \frac{\partial}{\partial \beta}\left(\frac{\partial \ln z}{\partial \beta}\right) + \left(\frac{\partial \ln z}{\partial \beta}\right)^2.$$

Dispersion of energy is

$$\overline{(\Delta E)^2} = \overline{E^2} - (\bar{E})^2 = \frac{\partial}{\partial \beta}\left(\frac{\partial \ln z}{\partial \beta}\right) + \left(\frac{\partial \ln z}{\partial \beta}\right)^2 - \left(-\frac{\partial \ln z}{\partial \beta}\right)^2 = \frac{\partial^2 \ln z}{\partial \beta^2} = -\frac{\partial}{\partial \beta}\bar{E} = kT^2 c_v.$$