Answer on Question #54126, Physics / Other

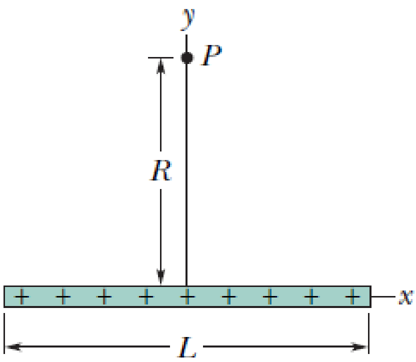
A straight, non-conducting plastic wire 8.50cm long carries a charge density of +175 nC/m distributed uniformly along its length. It is lying on a horizontal tabletop.

(a) Find the magnitude and direction of the Electric Field that is produced 6.0cm directly above its midpoint.

(b) If the wire is now bent into a circle lying flat on the table, find the magnitude and direction of the Electric field on a point 6.0cm directly above its center.

Solution:

(a)



Using the notation λ = q/L we note that the (infinitesimal) charge on an element dx of the rod contains charge dq = λ dx. By symmetry, we conclude that all horizontal field components (due to the dq's) cancel and we need only "sum" (integrate) the vertical components. Symmetry also allows us to integrate these contributions over only half the rod (0 ≤ x ≤ L/2) and then simply double the result. In that regard we note that sin θ = R/r where $r = \sqrt{x^2 + R^2}$.

Our element dq produces a differential electric field at point P, which is a distance r from the element. Treating the element as a point charge, we can express the magnitude of dE as

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}$$

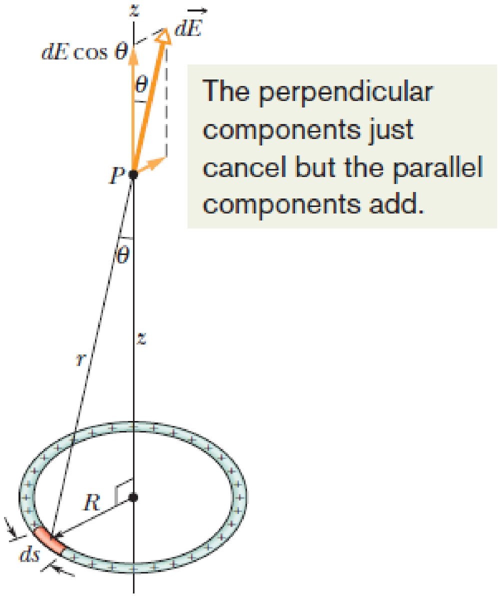
All field is

$$E = 2\int_0^{\frac{L}{2}} \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\sin\theta = \frac{2}{4\pi\varepsilon_0}\int_0^{\frac{L}{2}} \frac{\lambda dx}{(x^2+R^2)}\frac{y}{\sqrt{x^2+R^2}} = \frac{\lambda R}{2\pi\varepsilon_0}\int_0^{\frac{L}{2}}\frac{dx}{(x^2+R^2)^{3/2}}$$

$$= \frac{\lambda R}{2\pi\varepsilon_0}\frac{x}{R^2\sqrt{x^2+R^2}}\Bigg|_0^{L/2} = \frac{\lambda R}{2\pi\varepsilon_0}\frac{L/2}{R^2\sqrt{(L/2)^2+R^2}} = \frac{\lambda}{2\pi\varepsilon_0}\frac{L}{R\sqrt{L^2+4R^2}}$$

$$E = \frac{175*10^{-9}}{2*\pi*8.85*10^{-12}}*\frac{0.085}{0.06*\sqrt{0.085^2+4*0.06^2}} = 30318.2 = 3.032*10^4\ \mathrm{N/C}$$

(b)



The perpendicular components cancel and we need not consider them further. This leaves the parallel components; they all have the same direction, so the net electric field at P is their sum.

The parallel component of shown in figure has magnitude $dEcos\theta$.The figure also shows us that

$$\cos\theta = \frac{z}{r} = \frac{z}{\sqrt{z^2 + R^2}}$$

Let ds be the (arc) length of any differential element of the ring. Since λ is the charge per unit (arc) length, the element has a charge of magnitude

$$dq = \lambda\, ds$$

This differential charge sets up a differential electric field dE at point P, which is a distance r from the element.

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{\lambda ds}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda ds}{z^2 + R^2}$$

$$dE\cos\theta = \frac{z\lambda}{4\pi\varepsilon_0(z^2 + R^2)^{3/2}}ds$$

To add the parallel components produced by all the elements, we integrate around the circumference of the ring, from s = 0 to s = 2piR.

$$E = \int dE\cos\theta = \frac{z\lambda}{4\pi\varepsilon_0(z^2 + R^2)^{3/2}}\int_0^{2\pi R} ds = \frac{z\lambda(2\pi R)}{4\pi\varepsilon_0(z^2 + R^2)^{3/2}} = \frac{z\lambda L}{4\pi\varepsilon_0(z^2 + L^2/4\pi^2)^{3/2}}$$

$$E = \frac{0.06 * 175 * 10^{-9} * 0.085}{4 * \pi * 8.85 * 10^{-12} * (0.06^2 + 0.085^2/4\pi^2)^{3/2}} = 34490.4 \approx 3.45 * 10^4 \text{ N/C}$$

Answer: $3.032 * 10^4$ N/C; $3.45 * 10^4$ N/C