Answer on Question #54035, Physics Quantum Mechanics

Consider a quantum particle confined in a well of width a. If the particle is in its ground state calculate the quantity DxDp where ( ) 2 2 2 Dx = x − x and ( ) .

2 2 2 Dp = p – p

Solution:

The average value of P_X

$$\langle P_X \rangle = \int_0^a \psi_1^*(x)\hat{P}_X\psi_1(x)dx = \int_0^a \sqrt{\frac{2}{a}}\sin\left(\frac{\pi x}{a}\right)\left(-i\hbar\frac{\partial}{\partial x}\right)\sqrt{\frac{2}{a}}\sin\left(\frac{\pi x}{a}\right)dx = -\frac{2i\hbar}{a}\frac{\pi n}{a}\int_0^a \sin\left(\frac{\pi x}{a}\right)\cos\left(\frac{\pi x}{a}\right)dx =$$
$$-\frac{i\hbar\pi}{a^2}\int_0^a \sin\left(\frac{2\pi x}{a}\right)dx = 0$$

where a is the width of the potential well; $\psi_1 = \sqrt{\frac{2}{a}}\sin\left(\frac{\pi x}{a}\right)$ is the wave function of a one-dimensional potential well in ground state.

The average value of $P_X{}^2$

$$\langle P_X{}^2 \rangle = \int_0^a \psi_1^*(x)\hat{P}_X{}^2\psi_1(x)dx = \int_0^a \sqrt{\frac{2}{a}}\sin\left(\frac{\pi x}{a}\right)\left(-i\hbar\frac{\partial}{\partial x}\right)^2\sqrt{\frac{2}{a}}\sin\left(\frac{\pi x}{a}\right)dx =$$
$$\frac{2\hbar^2}{a}\left(\frac{\pi n}{a}\right)^2\int_0^a \sin\left(\frac{\pi x}{a}\right)\sin\left(\frac{\pi x}{a}\right)dx = \frac{\hbar^2\pi^2}{a^2}$$

The average value of $\hat{x}$

$$\langle x \rangle = \int_0^a \psi_1^*(x)\hat{x}\psi_1(x)dx = \int_0^a \sqrt{\frac{2}{a}}\sin\left(\frac{\pi x}{L}\right)x\sqrt{\frac{2}{L}}\sin\left(\frac{\pi x}{L}\right)dx = \frac{2}{a}\int_0^a \sin\left(\frac{\pi x}{a}\right)x\sin\left(\frac{\pi x}{a}\right)dx = a/2$$

The average value of x^2

$$\langle x^2\rangle = \int_0^a \psi_1^*(x)x^2\psi_1(x)\,dx = \int_0^a \sqrt{\frac{2}{a}}\sin\left(\frac{\pi x}{a}\right)x^2\sqrt{\frac{2}{a}}\sin\left(\frac{\pi x}{a}\right)dx = \frac{2}{a}\int_0^a \sin^2\left(\frac{\pi x}{a}\right)x^2dx = \frac{a^2}{6}\left(2-3/\pi^2\right)$$

Then

$$\Delta x\Delta p = \sqrt{\left(\frac{\hbar^2\pi^2}{a^2}-0\right)\left(\frac{a^2}{6}\left(2-3/\pi^2\right)-\frac{a^2}{4}\right)} = \frac{\hbar}{2\sqrt{3}}\sqrt{\pi^2-6}$$

Answer: $\Delta x\Delta p = \frac{\hbar}{2\sqrt{3}}\sqrt{\pi^2-6}$