Answer on Question #54034, Physics Quantum Mechanics

Obtain the expectation value of the potential energy 2 221V (x) = mw x of the onedimensional harmonic oscillator in the first excited state/ 21/ 212 22( ) 2 a x axeax _

Solution:

The expectation value of the potential energy

$$\langle\psi_1|V|\psi_1\rangle=\int_{-\infty}^{+\infty}\psi_1(x)V(x)\psi_1(x)dx=\int_{-\infty}^{+\infty}\frac{a^{3/2}\sqrt{2}}{\pi^{1/4}}x\exp\left[-\frac{a^2x^2}{2}\right]\frac{m\omega^2x^2}{2}\frac{a^{3/2}\sqrt{2}}{\pi^{1/4}}x\exp\left[-\frac{a^2x^2}{2}\right]dx=$$

$$\frac{2a^3}{\sqrt{\pi}}\int_{-\infty}^{+\infty}\frac{m\omega^2x^4}{2}\exp\left[-a^2x^2\right]dx=\frac{m\omega^2}{a^2\sqrt{\pi}}\int_{-\infty}^{+\infty}(ax)^4\exp\left[-a^2x^2\right]d(ax)=\left|\begin{array}{l}\xi=(ax)^2\\ d\xi=2axd(ax)=2\sqrt{y}\xi d(ax)\end{array}\right|$$

$$=\frac{m\omega^2}{a^2\sqrt{\pi}}\int_{-\infty}^{+\infty}\xi^2\exp[-\xi]\frac{d\xi}{2\sqrt{\xi}}=\frac{m\omega^2}{a^2\sqrt{\pi}}\int_{-\infty}^{+\infty}\xi^2\exp[-\xi]\frac{d\xi}{2\sqrt{\xi}}=\frac{m\omega^2}{2a^2\sqrt{\pi}}\int_{-\infty}^{+\infty}\xi^{3/2}\exp[-\xi]d\xi=$$

$$\frac{m\omega^2}{2a^2\sqrt{\pi}}\Gamma(5/2)=\frac{m\omega^2}{2a^2\sqrt{\pi}}\cdot\frac{3}{2}\cdot\frac{1}{2}\cdot\Gamma(1/2)=\frac{3m\omega^2}{8a^2\sqrt{\pi}}\cdot\sqrt{\pi}=\frac{3m\omega^2}{8a^2}$$

where $\Gamma(\xi)$ is the Gamma-function, $\psi_1(x)=\frac{a^{3/2}\sqrt{2}}{\pi^{1/4}}x\exp\left[-\frac{a^2x^2}{2}\right]$ is the wave function

Answer: $\langle\psi_1|V|\psi_1\rangle=\frac{3m\omega^2}{8a^2}$