Answer on question #53971, Physics / Relativity

Question A muon has a lifetime of $2 \cdot 10^{-6}$ s in its rest frame. It is created 100 km above the earth and moves towards it at a speed of $2.97 \cdot 10^8$ ms-1 .At what altitude does it decay ? What is the distance travelled by the muon during this time in its own frame of reference?

Solution Let us find life time in the Earth frame. Because of relativistic effect it will be

$$\Delta t_e = \frac{\Delta t}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{2 \cdot 10^{-6}}{\sqrt{1 - (2.97/2.99)^2}} \approx 17.32 \cdot 10^{-6} \, s$$

Having speed $2.97 \cdot 10^8$ muon will travel during this time

$$\Delta l_e = \Delta t_e \cdot v = 17.32 \cdot 10^{-6} \cdot 2.97 \cdot 10^8 = 5140 \, m$$

Hence, it will decay approximately at altitude 100 - 5 = 95 km.
Distance travelled in own frame:

$$\Delta l_{own} = l_e \sqrt{\sqrt{1 - \frac{v^2}{c^2}}} = 5140\sqrt{1 - (2.97/2.99)^2} \approx 593.5 \, m$$