Answer on Question #53641, Physics Mechanics Kinematics Dynamics

Prove that kinetic energy of a gas molecule depends only on its absolute temperature.

Answer:

Pressure is explained by kinetic theory as arising from the force exerted by molecules or atoms impacting on the walls of a container. Consider a gas of molecules, each of mass m, enclosed in a cuboidal container of volume $V = L^3$.

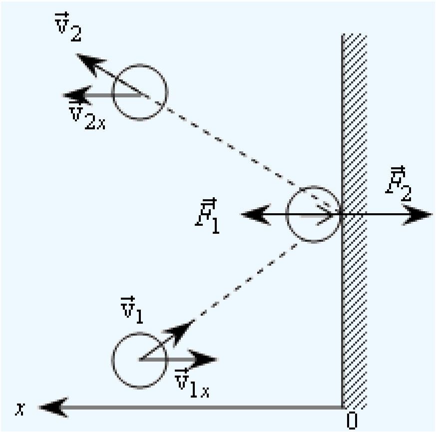

Fig.1

When a gas molecule collides with the wall of the container perpendicular to the x coordinate axis and bounces off in the opposite direction with the same speed , then the momentum lost by the particle and gained by the wall is:

$$\Delta p = p_x - (-p_x) = 2p = 2mv_x \quad (1)$$

where vx is the x-component of the initial velocity of the particle.

The particle impacts one specific side wall once every

$$\Delta t = \frac{2L}{v_x} \quad (2)$$

(where L is the distance between opposite walls).

The force due to this particle is:

$$f = \Delta p / \Delta t = m v_x^2 / L \tag{3}$$

The total force on the wall is

$$F = Nf = N m v_x^2 / L \tag{4}$$

where the bar denotes an average over the N particles. Since the assumption is that the particles move in random directions, we will have to conclude that if we divide the velocity vectors of all particles in three mutually perpendicular directions, the average value along each direction must be equal (though their proportions are arbitrary for individual particles). That is,

$$\overline{v_x^2} = \overline{v^2} / 3 \tag{5}$$

Then

$$F = N m v^2 / 3L \tag{6}$$

Therefore the pressure of the gas is

$$P = F / L^2 = N m v^2 / 3L^3 = N m v^2 / 3V \tag{7}$$

Rewriting the above result for the pressure as $PV = N m v^2 / 3$, we may combine it with the ideal gas law

$$PV = N k_B T \tag{8}$$

where k_B is the Boltzmann constant and T the absolute temperature defined by the ideal gas law.

So, the average kinetic energy of a molecule

$$K = \frac{m\overline{v^2}}{2} = \frac{3}{2}k_B T \tag{9}$$