Answer on Question #53396, Physics / Mechanics | Kinematics | Dynamics

A cannon with a muzzle speed of 1000 m/s is used to start an avalanche on a mountain slope. The target is 2000 m from the cannon horizontally and 800 m above the cannon. At what angle, above the horizontal should the cannon be fired?

Solution:

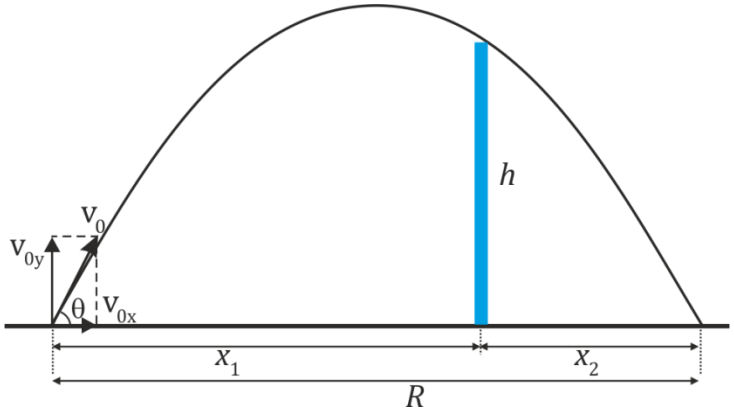
Given:

$x_1 = 2000$ m,

$h = 800$ m,

$v_0 = 1000$ m/s,

$\theta = ?$



Neglecting air resistance, the projectile is subject to a constant acceleration g=9.81 m/s^2, due to gravity, which is directed vertically downwards.

Equations related to trajectory motion (projectile motion) are given by

$$\text{Horizontal distance,} \quad x = v_{0x}t$$

$$\text{Vertical distance,} \quad y = v_{0y}t - \frac{1}{2}gt^2$$

where v_0 is the initial velocity.

From first equation,

$$t_1 = \frac{x_1}{v_{0x}} = \frac{x_1}{v_0 \cos\theta} = \frac{2000}{1000 \cos\theta} = \frac{2}{\cos\theta}$$

Substituting to second equation,

$$h = v_{0y}\frac{x_1}{v_0 \cos\theta} - \frac{1}{2}g\left(\frac{2}{\cos\theta}\right)^2$$

$$h = v_0 \sin\theta \frac{x_1}{v_0 \cos\theta} - \frac{1}{2}g\left(\frac{2}{\cos\theta}\right)^2$$

$$h = x_1 \tan\theta - \frac{1}{2}g\left(\frac{2}{\cos\theta}\right)^2$$

Applying trigonometric identities:

$$\frac{1}{\cos^2\theta} = \frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta} = \frac{\sin^2\theta}{\cos^2\theta} + 1 = \tan^2\theta + 1$$

$$h = x_1 \tan\theta - 2g\,(\tan^2\theta + 1)$$

$$800 = 2000\tan\theta - 19.6\,(\tan^2\theta + 1)$$

Substituting

$$z = \tan\theta$$

$$800 = 2000z - 19.6\,(z^2 + 1)$$

$$19.6z^2 - 2000z + 819.6 = 0$$

Solutions of quadratic equation:

$$z_1 = 0.411459$$
$$z_2 = 101.629$$

Back to $tan(\theta) = z$

$$\tan(\theta_1) = 0.411459$$
$$\tan(\theta_2) = 101.629$$

$$\theta_1 = \tan^{-1}(0.411459) = 22.37°$$
$$\theta_2 = \tan^{-1}(101.629) = 89.44°$$

Answer: 22.37° or 89.44°