Answer on Question #53365, Physics / Other

Water circulates throughout the house in a hot-water heating system, if the water pumped at a speed of 0.40 m/s through a 4.2 cm diameter pipe in the basement under a pressure of 2.8 atm, what will be the flow speed and pressure in a 2.6 cm diameter pipe on the second floor 5.0m above?

Solution:

The equation of continuity states that for an incompressible fluid flowing in a tube of varying cross-section, the mass flow rate is the same everywhere in the tube. The mass flow rate is simply the rate at which mass flows past a given point, so it's the total mass flowing past divided by the time interval. The equation of continuity can be reduced to:

$$A_1 v_1 = A_2 v_2$$

where A is the cross-sectional area, *v* is the velocity. The subscripts 1 and 2 refer to the initial conditions at ground level and the final conditions inside the nozzle, respectively.

$$A = \pi r^2$$

Given:

$v_1 = 0.40 \text{ m/s}$,

$r_1 = 0.021 \text{ m}$,

$r_2 = 0.013 \text{ m}$,

$v_2 = ? \text{ m/s}$,

Thus,

$$\pi r_1^2 v_1 = \pi r_2^2 v_2$$

So,

$$v_2 = \left(\frac{r_1}{r_2}\right)^2 v_1$$

$$v_2 = \left(\frac{0.021}{0.013}\right)^2 \cdot 0.40 = \mathbf{1.04\ m/s}$$

Bernoulli's equation is one of the most important/useful equations in fluid mechanics. It may be written,

$$p + \frac{1}{2}\rho v^2 + \rho g h = constant$$

where *p* is the pressure, ρ is the density, *v* is the velocity, *h* is the elevation, and *g* is the gravitational acceleration.

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

The subscripts 1 and 2 refer to the initial conditions at ground level and the final conditions inside the nozzle, respectively.

Given:

$\rho = 1000 \text{ kg/m}^3$,

$v_1 = 0.40 \text{ m/s}$,

$h_1 = 0,$
$p_1 = 2.8 \text{ atm}$
$h_2 = 5.0 \text{ m},$
$p_2 = ?,$

$$p_2 = p_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 - \frac{1}{2}\rho v_2^2 - \rho g h_2$$

$$p_2 = p_1 + \rho\left(\frac{1}{2}v_1^2 - \frac{1}{2}v_2^2 - g h_2\right)$$

Thus,

$$p_2 = 2.8 * 10^5 + 1000 \cdot \left(\frac{1}{2} * 0.4^2 - \frac{1}{2} * 1.04^2 - 9.81 \cdot 5.0\right) = 2.8 * 10^5 - 49510.8 =$$
$$= 230489.2 \text{ Pa} = \mathbf{2.3\ atm}$$

Answer: $v_2 = \mathbf{1.04\ m/s};\ \ p_2 = \mathbf{2.3\ atm}$