Answer on Question#53335 - Physics - Mechanics - Kinematics - Dynamics

If h_1 and h_2 are the greatest height of the projectile in two paths for a give value of range, then what is the horizontal range?

Solution:

Let the initial speed of the projectile be v_0 and the angle at which it's fired – α. The greatest height is then given by

$$h = \frac{v_0^2 \sin^2 \alpha}{2g},$$

and, the range is

$$L = \frac{v_0^2}{g} 2 \sin \alpha \cos \alpha$$

From the first equation we obtain

$$\sin \alpha = \sqrt{\frac{2gh}{v_0^2}}$$

Therefore

$$\cos \alpha = \sqrt{1 - \frac{2gh}{v_0^2}}$$

Substituting these into the second equation we obtain

$$L = \frac{v_0^2}{g} 2 \sqrt{\frac{2gh}{v_0^2}} \sqrt{1 - \frac{2gh}{v_0^2}}$$

$$\frac{L^2 g^2}{v_0^2} = 8gh \left(1 - \frac{2gh}{v_0^2}\right)$$

$$h^2 - \frac{v_0^2}{2g} + \frac{L^2}{16} = 0$$

According to the Vieta's formulas (h_1 and h_2 – are roots of this equation) we obtain

$$h_1 \cdot h_2 = \frac{L^2}{16}$$

Therefore

$$L = 4\sqrt{h_1 \cdot h_2}$$

Answer: $4\sqrt{h_1 \cdot h_2}$.