**Answer on Question #53334, Physics / Mechanics | Kinematics | Dynamics**

Q. Why work done is vector quantity? But it is dot product of two vector quantities force and displacement

**Solution:**

In science we define the work done by a force as the magnitude of the force multiplied by the distance it moves along its line of action or by the component of the magnitude of the force in a given direction multiplied by the distance moved in that direction. Work is a scalar quantity and the product obtained when force is multiplied by displacement is called the scalar product.

The work done defined as a dot-product (or scalar product) of force and displacement, both of which are vectors.

A scalar product of two vectors gives a scalar result.

$$dW = \vec{F} \cdot \vec{S} = \|F\|\|S\|cos\theta$$

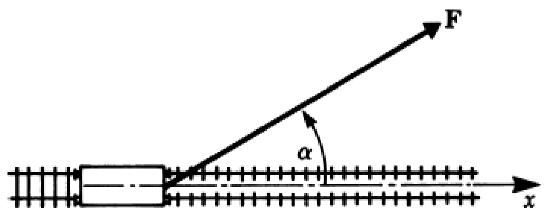
(θ being the angle between the vectors).

We have no direction, only magnitude.

A force can do work on a body even displacing at an angle to the direction of force (θ).

It should be noted that when θ is $90°$, the result will be zero ($cos90° = 0$). When force and displacement are perpendicular, the force does no work on the body.

We can consider the example of the Scalar Product. First, we consider a carriage running on rails. It moves in the s-direction under the application of a force F which acts at an angle $\alpha$ to the direction of travel. The information is provided in Figure 1.



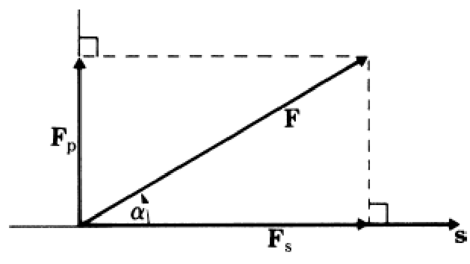




Figure 1

In given task we need to find the work done by the force when the carriage moves through a distance s in the s-direction.

In order to study the action of the force F on the carriage we resolve it into two components: one along the rails (in the s-direction), and one perpendicular to the rails, i.e. $F_s$ and $F_p$ respectively. $F_s$, $F_p$ and $s$are vector quantities; the work is, by definition, the product of the force along the direction of motion and the distance moved. In this case, it is the product of $F_s$ and $s$. It follows also from the definition that the work done by $F_p$ is zero since there is no displacement in that direction. Furthermore, if the rails are horizontal then the motion of the carriage and the work done is not influenced by gravity, since it acts in a direction perpendicular to the rails.

If W is the work done then $W = F \cdot \cos\alpha \cdot s$ or $F \cdot s \cdot \cos\alpha$ in magnitude.

Since work is a scalar quantity the product of the two vectors is called a scalar product or dot product, because one way of writing it is with a dot between the two vectors:

$$W = F_s \cdot s$$

Where

$$|F_s| = |F| \cos\alpha$$

It is also referred to as the inner product of two vectors. Generally, if a and b are two vectors their inner product is written $a \cdot b$. Thus, the inner or scalar product of two vectors is equal to the product of their magnitude and the cosine of the angle between their directions:

$$a \cdot b = a \cdot b \cos\alpha$$

The scalar product of two vectors a and b is equal to the product of the magnitude of vector a with the projection of b on a.

The concept of "work" as the name of a physical quantity is revealed in two ways: as the mechanical work (force work), depending on the force vectors and movement, and how the thermodynamic work - the amount of energy transmitted or received by the system by changing its external parameters. However, in physics and applied the concept of "field work", but it is treated as a "work force field."

The geometrical figure of the day will be a curve. If we have a function defined on a curve we can break up the curve into tiny line segments, multiply the length of the line segments by the function value on the segment and add up all the products.

The main application of line integrals is finding the work done on an object in a force field. If an object is moving along a curve through a force field F, then we can calculate the total work done by the force field by cutting the curve up into tiny pieces. The work done W along each piece will be approximately equal to

$$\mathrm{dW} = \mathrm{F} \cdot \mathrm{Tds}$$

Now, it should be noted the following

$$\mathrm{T} = \frac{\mathrm{r}'(\mathrm{t})}{\|\mathrm{r}'(\mathrm{t})\|}$$

And the ds equal to $\|\mathrm{r}'(\mathrm{t})\|\mathrm{dt}$

Thus, we can finally note the following

$$dW = F \cdot r'(t)dt$$

As usual, we add up all the small pieces of work and take the limit as the pieces get small to end up with an integral.

Thus, we can note the definition of the work done.

Let F be a vector field and C be a curve defined by the vector valued function r. Then the work done by F on an object moving along C is given by

Work done= $\int_C F \cdot dr = \int_a^b F\big(x(t), y(t), z(t)\big) \cdot r'(t)dt$

Now, we can consider the example in order to illustrate the work done.

We have $F(x, y, z) = xi + 3xyj - (x + z)k$, in given problem we need to find the work done by the vector field on a particle moving along the line segment that goes from (1,4,2) to (0,5,1)

First, we have to parameterize the curve. We obtain the following

$$r(t) =< 1{,}4{,}2 > +[< 0{,}5{,}1 > - < 1{,}4{,}2 >]t =< 1 - t, 4 + t, 2 - t >$$

and

$$r'(t) = -i + j - k$$

Then, we take the dot product and get the following result

$$F \cdot r'(t) = -x + 3xy + x + z = 3xy + z = 3(1 - t)(4 + t) + (2 - t) = -3t^2 - 10t + 14$$

Finally, we integrate the obtain expression.

$$\int_0^1 (-3t^2 - 10t + 14)dt = [-t^3 - 5t^2 + 14t]_0^1 = -1 - 5 + 14 - 0 = 8$$

Thus, we can conclude that work done by a force field on an object moving along a curve depends on the direction that the object goes. In fact the opposite direction will produce the negative of the work done in the original direction.