Answer on Question#53285 - Physics - Electromagnetism

The Fraunhofer diffraction pattern of a circular aperture (of radius $a = 0.5 \text{ mm}$) is observed on the local plane of a convex lens of focal length $F = 20 \text{ cm}$. Calculate the radii of the first and the second dark rings. Assume $\lambda = 5.5 \times 10^{-8}\text{cm}$.

Solution:

The intensity of the light in the focal plane is given by

$$I(\theta) = I_0 \left(2 \frac{J_1(ka \sin\theta)}{ka \sin\theta} \right)^2,$$

where I_0 – is the intensity in the plane of the circuit, θ – is the angle between the optical axis of the lens and the direction of the point under consideration in the focal plane, and $k = \frac{2\pi}{\lambda}$ – is the wavenumber.

$\sin\theta$ can be rewritten in terms of the radius of the dark ring and the focal length as follows

$$\sin\theta = \frac{R}{F}$$

Therefore

$$I(R) = I_0 \left(2 \frac{J_1\left(\frac{kaR}{F}\right)}{\frac{kaR}{F}} \right)^2$$

The dark ring occurs when $I(R) = 0$. First two dark rings are defined by the 2nd and the 3rd zeros of the Bessel function J_1. These zeros are

$$\mu_2 = 3.83, \qquad \mu_3 = 7.01$$

Therefore

$$\frac{kaR_1}{F} = \mu_2, \qquad \frac{kaR_2}{F} = \mu_3$$

$$R_1 = \frac{F\mu_2}{ka} = \frac{F\lambda\mu_2}{2\pi a} = \frac{20 \text{ cm} \cdot 5.5 \times 10^{-8}\text{cm} \cdot 3.83}{2\pi \cdot 0.5 \text{ mm}} = 0.13 \text{ μm}$$

$$R_2 = \frac{F\mu_3}{ka} = \frac{F\lambda\mu_3}{2\pi a} = \frac{20 \text{ cm} \cdot 5.5 \times 10^{-8}\text{cm} \cdot 7.01}{2\pi \cdot 0.5 \text{ mm}} = 0.25 \text{ μm}$$

Answer: $R_1 = 0.13$ μm, $R_2 = 0.25$ μm.