

Answer on Question #53248, Physics / Mechanics | Kinematics | Dynamics

An 800kg roller coaster car is launched horizontally from a giant spring ($k = 35 \text{ kN/m}$), rolls 10 m and then goes through a vertical loop of radius 7 m. If for every meter the car rolls on the track, thermal energy increases by 800 J, would compressing the spring 3 m give the car enough energy to safely get to the top of the loop?

Solution:

At top point Newton's 2nd law gives:

$$N + mg = \frac{mv^2}{R}$$

The point where the car is about to leave the track is when $N = 0$. So at top point,

$$mg = \frac{mv^2}{R}$$

or

$$v = \sqrt{gR}$$

Energy is always conserved. If there's friction, mechanical energy is "lost", but surfaces heat up. Mechanical energy has been transformed into heat energy.

Mechanical energy, $K+U$, is not conserved if there's friction, but total energy, $K+U+\text{thermal}$ is conserved.

The initial energy is

$$E_i = \frac{1}{2}kx^2 = \frac{1}{2} * 35 * 3^2 = 157.5 \text{ kJ}$$

The final energy at top of loop is

$$E_f = mgh + \frac{1}{2}mv^2 = mg2R + \frac{1}{2}mgR = \frac{5}{2}mgR$$

$$E_f = \frac{5}{2} * 800 * 9.8 * 7 = 137.2 \text{ kJ}$$

The thermal energy is

$$E_{th} = 0.8L = 0.8 * (10 + \pi R) = 0.8 * (10 + \pi * 7) = 25.6 \text{ kJ}$$

For energy,

$$E_f + E_{th} = 137.2 + 25.6 = 162.8 \text{ kJ}$$

Thus,

$$E_i < E_f + E_{th}$$

Answer: The car **will not** get to the top of the loop.