Answer on Question #53235, Physics / Optics

The Fraunhofer diffraction pattern of a circular aperture (of radius 0.5 mm) is observed on the focal plane of a convex lens of focal length 20cm. Calculate the radii of the first and the second dark rings. Assume λ = 5.5x10^{-5} cm.

Solution:

The amplitude distribution for diffraction due to a **circular aperture** forms an intensity pattern with a bright central band surrounded by concentric circular bands of rapidly decreasing intensity (Airy pattern). The 1st maximum is roughly 1.75% of the central intensity. 84% of the light arrives within the central peak called the Airy disk.

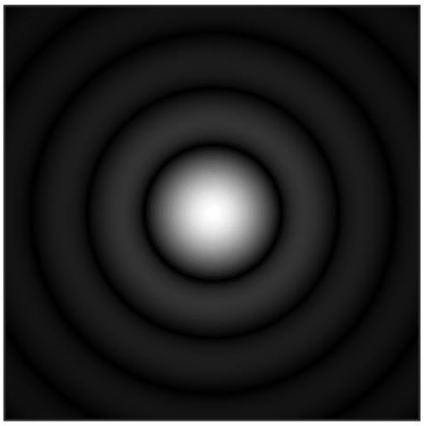

Fig.. An image of an **Airy disk**.

Far away from the aperture, the angle at which the first minimum occurs, measured from the direction of incoming light, is given by the approximate formula:

$$\sin\theta \approx 1.22\frac{\lambda}{d}$$

or, for small angles, simply

$$\theta \approx 1.22\frac{\lambda}{d}$$

where θ is in radians, λ is the wavelength of the light and d is the diameter of the aperture.

We have $\lambda = 5.5 * 10^{-5}$ cm and $d = 2r = 2 * 0.5 = 1$ mm $= 0.1$ cm

Thus,

$$\theta \approx 1.22 * \frac{5.5 * 10^{-5}}{0.1} = 67.1 * 10^{-5} \text{ rad}$$

Hence, the radius of the first dark ring

$$r_1 = f\theta = 20 * 67.1 * 10^{-5} = 0.01342 \text{ cm} \approx 0.13 \text{ mm}$$

Similarly for the second dark ring

$$\theta_2 \approx \sin\theta_2 = \frac{2.23\lambda}{d} = \frac{2.23 * 5.5 * 10^{-5}}{0.1} = 122.65 * 10^{-5} \text{ rad}$$

$$r_2 = f\theta_2 = 20 * 122.65 * 10^{-5} = 0.02453 \text{ cm} \approx 0.25 \text{ mm}$$

Answer: $r_1 \approx 0.13$ mm; $r_2 \approx 0.25$ mm.