# Answer on Question#52857 - Physics - Atomic Physics

An object of mass, $m = 5\text{kg}$, at the origin has a velocity of $\boldsymbol{v_i} = (12\boldsymbol{i} - 18\boldsymbol{j})\frac{\text{m}}{\text{s}}$ at $t = 0$. It is accelerated at a constant rate for 5 seconds after which it has a velocity of $\boldsymbol{v_f} = (5\boldsymbol{i} - 7\boldsymbol{j})\frac{\text{m}}{\text{s}}$.

1. What is the magnitude of the resultant force acting on the object during this time interval?
2. How far is it from the origin after 3 seconds?
3. What is the speed after 3 seconds?

Solution:

1. Action of the force on the object changes its momentum as follows

$$\int_{t_i}^{t_f} \boldsymbol{F} dt = \Delta \boldsymbol{p},$$

where $t_i$, $t_f$ – are initial and final times, $\boldsymbol{F}$ – is the resultant force, $\boldsymbol{p} = m\boldsymbol{v}$ – is the momentum of the object. Since the $\boldsymbol{F}$ is constant (object is accelerated at a constant rate), $t_i = 0$, $t_f = 5\text{s}$, $\Delta\boldsymbol{p} = m(\boldsymbol{v_f} - \boldsymbol{v_i})$, we obtain

$$\boldsymbol{F}(t_f - t_i) = m(\boldsymbol{v_f} - \boldsymbol{v_i})$$

$$F = \frac{m(\boldsymbol{v_f} - \boldsymbol{v_i})}{t_f - t_i} = \frac{5\text{kg} \cdot \left(5\boldsymbol{i} - 7\boldsymbol{j} - (12\boldsymbol{i} - 18\boldsymbol{j})\right)\frac{\text{m}}{\text{s}}}{5\text{s} - 0} = (-\boldsymbol{7i} + 11\boldsymbol{j})\text{N}$$

The magnitude of the force:

$$|\boldsymbol{F}| = \sqrt{(-7)^2 + (11)^2}\text{N} = \sqrt{170}\text{N} = 13\text{N}$$

2. The displacement is given by

$$\boldsymbol{s}(t) = \boldsymbol{s_0} + \boldsymbol{v_i} \cdot t + \frac{\boldsymbol{a} \cdot t^2}{2},$$

where $\boldsymbol{s_0} = \boldsymbol{0}$ – is the initial position of the object, $\boldsymbol{a} = \frac{\boldsymbol{F}}{m} = \frac{(-7\boldsymbol{i}+11\boldsymbol{j})\text{N}}{5\text{kg}} = (-1.4\boldsymbol{i} + 2.2\boldsymbol{j})\frac{\text{m}}{\text{s}^2}$ – is the acceleration of the object, $t$ – is the elapsed time.

Since $t = 3\text{s}$, we obtain

$$\boldsymbol{s}(3\text{s}) = (12\boldsymbol{i} - 18\boldsymbol{j})\frac{\text{m}}{\text{s}} \cdot 3\text{s} + \frac{(-1.4\boldsymbol{i} + 2.2\boldsymbol{j})\frac{\text{m}}{\text{s}^2} \cdot (3\text{s})^2}{2} = (29.7\boldsymbol{i} + 63.9\boldsymbol{j})\text{m}$$

The magnitude of $\boldsymbol{s}$ is:

$$|\boldsymbol{s}| = \sqrt{(29.7)^2 + (63.9)^2}\text{m} = \sqrt{4965.3}\text{m} = 70.5\text{m}$$

3. The dependence of velocity on time is given by

$$\boldsymbol{v}(t) = \boldsymbol{v_i} + \boldsymbol{a} \cdot t,$$

where $t$ – is elapsed time. Since $t = 3\text{s}$, we obtain

$$\boldsymbol{v}(3\text{s}) = (12\boldsymbol{i} - 18\boldsymbol{j})\frac{\text{m}}{\text{s}} + (-1.4\boldsymbol{i} + 2.2\boldsymbol{j})\frac{\text{m}}{\text{s}^2} \cdot 3\text{s} = (7.8\boldsymbol{i} - 11.4\boldsymbol{j})\frac{\text{m}}{\text{s}}$$

The speed is given by the magnitude of this vector

$$v(3\text{s}) = |\boldsymbol{v}(3\text{s})| = \sqrt{(7.8)^2 + (-11.4)^2}\frac{\text{m}}{\text{s}} = \sqrt{190.8}\frac{\text{m}}{\text{s}} = 13.8\frac{\text{m}}{\text{s}}$$

Answer:

1. $13\mathrm{N}$
2. $70.5\mathrm{m}$
3. $13.8\,\frac{\mathrm{m}}{\mathrm{s}}$