Answer on Question#52853 - Physics - Mechanics - Kinematics - Dynamics

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5/ (20 pts) A yo-yo of mass M, rotational inertia I, and inner and outer radii r and R, is gently pulled by a string with tension T on a rough floor as shown in Figure 1. Calculate the linear acceleration of the yo-yo.

$a_{com,x} = \frac{T + \frac{C}{I}}{M + \frac{1}{Tr}}$

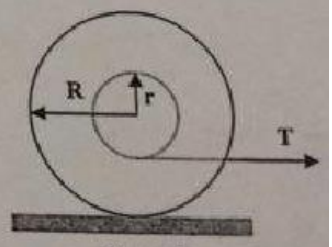

Figure 1

Solution:

Equation of motion, written about the point of contact with the surface, is given by

$$I_c \varepsilon = T \cdot (R - r),$$

where I_c – is the moment of inertia about the point of contact, ε – is the angular acceleration, $T \cdot (R - r)$ – is the torque of the force $\boldsymbol{T}$ about point of the contact. According to the parallel axis theorem the moment of inertia about the point of contact is given by

$$I_c = I + M \cdot R^2,$$

Therefore, the angular acceleration is given by

$$\varepsilon = \frac{T \cdot (R - r)}{I_c} = \frac{T \cdot (R - r)}{I + M \cdot R^2}$$

The linear acceleration a and the angular acceleration are connected by the following relation

$$a = \varepsilon \cdot R$$

Therefore,

$$a = \frac{T \cdot (R - r)}{I + M \cdot R^2} \cdot R = \frac{T \cdot R \cdot (R - r)}{I + M \cdot R^2}$$

Answer: $\frac{T \cdot R \cdot (R - r)}{I + M \cdot R^2}$.