Answer on Question 52652, Physics, Other

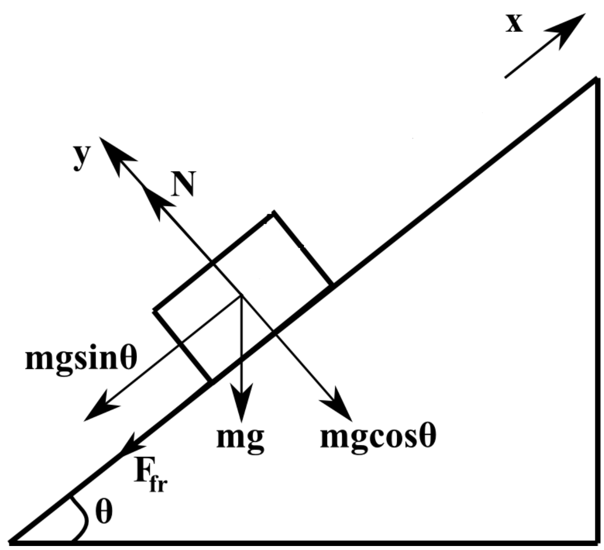
Question:

A $2kg$ box is projected with an initial speed of $3\,m/s$ up a rough plane inclined at 60° above horizontal. The coefficient of kinetic friction is 0.3.

a) What is the energy dissipated by friction as the box slides up the plane?

b) What is the speed of the box when it again reaches its initial position?

Solution:



Let's write all forces that acts on a box:

$$m\vec{g} + \vec{N} + \overrightarrow{F_{fr}} = m\vec{a}$$

Then projected the forces on axis x and y:

$$-mgsin\theta - F_{fr} = ma,$$

$$N - mgcos\theta = 0.$$

By the definition, the friction force is $F_{fr} = \mu_k N = \mu_k mgcos\theta$, and we can find the acceleration of the box from the first equation:

$$-mgsin\theta - \mu_k mgcos\theta = ma,$$

$$a = -g(sin\theta + \mu_k cos\theta).$$

a) The energy dissipated by friction as the box slides up the plane is equal to the work done on the box by the friction force as the box slides up the plane:

$$W_{fr} = F_{fr}s = \mu_k mgscos\theta,$$

where, s is the distance, that the box slides up the plane before it stops momentarily (when $v = 0$).

We can find s from the kinematic equation:

$$v^2 = v_0^2 + 2as.$$

Because $v = 0$ we get:

$$s = -\frac{v_0^2}{2a} = \frac{v_0^2}{2g(sin\theta + \mu_k cos\theta)} = \frac{\left(3\frac{m}{s}\right)^2}{2 \cdot 9.8\frac{m}{s^2} \cdot (sin60^{\circ} + 0.3 \cdot cos60^{\circ})} = 0.452m.$$

As we know s we can find the energy dissipated by friction as the box slides up the plane:

$$W_{fr} = F_{fr}s = \mu_k mgscos\theta = 0.3 \cdot 2kg \cdot 9.8\frac{m}{s^2} \cdot 0.452m \cdot cos60^{\circ} = 1.33J.$$

b) In order to find the speed of the box when it again reaches its initial position we use the law of conservation of energy:

$$KE + PE = W_{grav.force} - W_{fr},$$

where, KE and PE is the kinetic energy and the potential energy of the box at initial position, $W_{grav.force}$ is the work done on the box by the gravitational force, W_{fr} is the work done on the box by the friction force as the box slides down to its initial position, respectively.

Let's obtain the work done on the box by the gravitational force:

$$W_{grav.force} = \int_{start}^{end} \overrightarrow{F_g} d\vec{s},$$

where, $d\vec{s}$ is a vector along inclined rough plane in the direction of motion, and $\overrightarrow{F_g}$ is the gravitational force pointing down vertically.

So, we obtain:

$$\overrightarrow{F_g} d\vec{s} = mgdscos\left(\frac{\pi}{2} + \theta\right) = -mgdssin\theta,$$

$$W_{grav.force} = \int_s^0 \overrightarrow{F_g} d\vec{s} = \int_s^0 -mgsin\theta ds = mgssin\theta.$$

Then, we can substitute $W_{grav.force}$ to the equation for the law of conservation of energy and obtain v:

$$\frac{1}{2}mv^2 = mgssin\theta - \mu_k mgscos\theta,$$

$$v = \sqrt{2gs(sin\theta - \mu_k cos\theta)} = \sqrt{2 \cdot 9.8\frac{m}{s^2} \cdot 0.452m \cdot (sin60^{\circ} - 0.3 \cdot cos60^{\circ})} =$$

$$= 2.52\frac{m}{s}.$$

Answer:

a) $W_{fr} = 1.33J$.

b) $v = 2.52\frac{m}{s}$.