Answer on Question #52510-Physics-Other

An engine using 1.00 mol of an ideal gas initially at a volume of 24.6 L and a temperature of 400 K

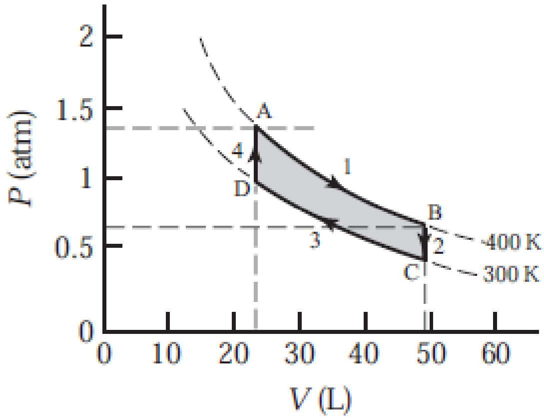
performs a cycle consisting of four steps: (1)an isothermal expansion at 400 K to twice its initial volume,(2)cooling at constant volume to a temperature of 300 K (3) an isothermal compression to its original volume, and (4)heating at constant volume to its original temperature of 400 K. Assume that Cv = 21.0 J/K.

Sketch the cycle on a PV diagram and find its efficiency.

Solution

We can find the efficiency of the cycle by finding the work done by the gas and the heat that enters the system per cycle.

The PV diagram of the cycle is shown to the right. A, B, C, and D identify the four states of the gas and the numerals 1, 2, 3, and 4 represent the four steps through which the gas is taken.



Express the efficiency of the cycle:

$$\varepsilon = \frac{W}{Q_h} = \frac{W_1 + W_2 + W_3 + W_4}{Q_{h,1} + Q_{h,2} + Q_{h,3} + Q_{h,4}}$$

Because steps 2 and 4 are constant volume processes, $W_2 = W_4 = 0$:

$$\varepsilon = \frac{W}{Q_h} = \frac{W_1 + 0 + W_3 + 0}{Q_{h,1} + Q_{h,2} + Q_{h,3} + Q_{h,4}}$$

Because the internal energy of the gas increases in step 4 while no work is done, and because the internal energy does not change during step 1 while work is done by the gas, heat enters the system only during these processes:

$$\varepsilon = \frac{W}{Q_h} = \frac{W_1 + W_3}{Q_{h,1} + Q_{h,4}} \qquad (1)$$

The work done during the isothermal expansion (1) is given by:

$$W_1 = nRT\ln\left(\frac{V_B}{V_A}\right)$$

The work done during the isothermal compression (3) is given by:

$$W_3 = nRT_c \ln\left(\frac{V_D}{V_C}\right)$$

Because there is no change in the internal energy of the system during step 1, the heat that enters the system during this isothermal expansion is given by:

$$Q_1 = W_1 = nRT_h \ln\left(\frac{V_B}{V_A}\right)$$

The heat that enters the system during the constant-volume step 4 is given by:

$$Q_4 = C_V \Delta T = C_V (T_h - T_c)$$

Substituting in equation (1) yields:

$$\varepsilon = \frac{nRT_h \ln\left(\frac{V_B}{V_A}\right) + nRT_c \ln\left(\frac{V_D}{V_C}\right)}{nRT_h \ln\left(\frac{V_B}{V_A}\right) + C_V (T_h - T_c)}$$

Noting the $\frac{V_B}{V_A} = 2$; and $\frac{V_D}{V_C} = \frac{1}{2}$, substitute and simplify to obtain:

$$\varepsilon = \frac{T_h \ln(2) + T_c \ln\left(\frac{1}{2}\right)}{T_h \ln(2) + \frac{C_V}{nR}(T_h - T_c)} = \frac{T_h \ln(2) - T_c \ln(2)}{T_h \ln(2) + \frac{C_V}{nR}(T_h - T_c)} = \frac{T_h - T_c}{T_h + \frac{C_V}{nR\ln(2)}(T_h - T_c)}$$

Substitute numerical values and evaluate ε:

$$\varepsilon = \frac{400\ \mathrm{K} - 300\ \mathrm{K}}{400\ \mathrm{K} + \frac{21.0\,\frac{\mathrm{J}}{\mathrm{K}}}{(1.00\ \mathrm{mol})\left(8.314\,\frac{\mathrm{J}}{\mathrm{mol}\cdot\mathrm{K}}\right)\ln(2)}(400\ \mathrm{K} - 300\ \mathrm{K})} = \boxed{13.1\%}$$