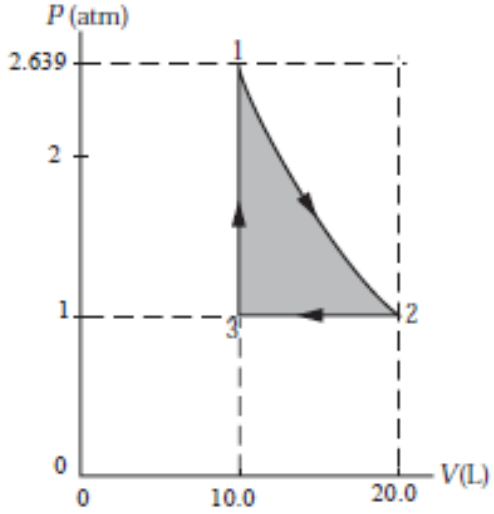
Answer on Question #52509-Physics-Other

The working substance of an engine is 1.00 mol of a diatomic ideal gas. The engine operates in a cycle consisting of three steps: (1) an adiabatic expansion from an initial volume of 10.0 L to a pressure of 1.00 atm and a volume of 20.0 L, (2) a compression at constant pressure to its original volume of 10.0 L, and (3) heating at constant volume to its original pressure. Take γ = 1.4.

Sketch the cycle on a PV diagram and find its efficiency.

Solution

The three steps in the process are shown on the PV diagram. We can find the efficiency of the cycle by finding the work done by the gas and the heat that enters the system per cycle.



The pressures and volumes at the end points of the adiabatic expansion are related according to:

$$P_1V_1^{\gamma} = P_2V_2^{\gamma} \Rightarrow P_1 = \left(\frac{V_2}{V_1}\right)^{\gamma} P_2$$

Substitute numerical values and evaluate P_1:

$$P_1 = \left(\frac{20.0\,\text{L}}{10.0\,\text{L}}\right)^{1.4} (1.00\,\text{atm}) = 2.639\,\text{atm}$$

Express the efficiency of the cycle:

$$\varepsilon = \frac{W}{Q_h} \qquad (1)$$

No heat enters or leaves the system during the adiabatic expansion:

$$Q_{12} = 0$$

Find the heat entering or leaving the system during the isobaric compression:

$$\begin{aligned} Q_{23} &= C_V \Delta T_{23} = \tfrac{7}{2} R \Delta T_{23} = \tfrac{7}{2} P \Delta V_{23} \\ &= \tfrac{7}{2}(1.00\,\text{atm})(10.0\,\text{L} - 20.0\,\text{L}) \\ &= -35.0\,\text{atm}\cdot\text{L} \end{aligned}$$

Find the heat entering or leaving the system during the constant volume process:

$$\begin{aligned} Q_{31} &= C_V \Delta T_{31} = \tfrac{5}{2} R \Delta T_{31} = \tfrac{5}{2} \Delta P V_{31} \\ &= \tfrac{5}{2}(2.639\,\text{atm} - 1.00\,\text{atm})(10.0\,\text{L}) \\ &= 41.0\,\text{atm}\cdot\text{L} \end{aligned}$$

Apply the 1st law of thermodynamics to the cycle (cycle $E_{int,cycle} = 0$) to obtain:

$$\begin{aligned} W_{on} &= \Delta E_{int} - Q_{in} = -Q_{in} \\ &= Q_{12} + Q_{23} + Q_{31} \\ &= 0 - 35.0\,\text{atm}\cdot\text{L} + 41.0\,\text{atm}\cdot\text{L} \\ &= 6.0\,\text{atm}\cdot\text{L} \end{aligned}$$

Substitute numerical values in equation (1) and evaluate ε:

$$\varepsilon = \frac{6.0\,\text{atm}\cdot\text{L}}{41\,\text{atm}\cdot\text{L}} = \boxed{15\%}$$