Answer on Question #52508-Physics-Other

a/ The pressure exerted on an ideal gas at 2.00 atm and 300 K is reduced suddenly to 1.00 atm while heat is transferred to maintain the initial temperature of 300 K. Calculate the heat, the work, and the change in internal energy for this process.

b/ Suppose the pressure change in question 1/ is carried out reversibly as well as isothermally. Calculate the heat, the work, and the change in internal energy for this process and compare the answers to those in the irreversible expansion in one step.

c/ Suppose the pressure change in question 1/ is carried out in the opposite direction (compression). Calculate the heat, the work, and the change in internal energy for this process in both a reversible and irreversible way.

Solution

a) For an ideal gas undergoing an isothermal process, ΔU = 0. The work done at constant external pressure is $w = -P_{ex}\,\Delta V$, where $\Delta V = V_{final} - V_{initial}$.

$$V_{initial} = \frac{nRT}{P_{initial}} = \frac{(1\,mol)(0.082\,06\,L\cdot atm\cdot K^{-1}\cdot mol^{-1})(300\,K)}{2.00\,atm} = 12.3\,L.$$

$$V_{final} = \frac{nRT}{P_{final}} = \frac{(1\,mol)(0.082\,06\,L\cdot atm\cdot K^{-1}\cdot mol^{-1})(300\,K)}{1.00\,atm} = 24.6\,L.$$

$$\Delta V = V_{final} - V_{initial} = 24.6\,L - 12.3\,L = 12.3\,L\,(system\ expands)$$

$$w = -P_{ex}\,\Delta V = -(1.00\,atm)(12.3\,L) = -12.3\,L\cdot atm = \\ = (-12.3\,L\cdot atm)(101.325\,J\cdot L^{-1}\cdot atm^{-1}) = -1.25\cdot 10^3\,J.$$

Work is done by the system on the surroundings.

$$q = \Delta U - w = 0 - (-1.25\cdot 10^3\,J) = +1.25\cdot 10^3\,J.$$

Heat is absorbed by the system from the surroundings.

b) The internal energy of an ideal gas depends only on temperature, and $\Delta U = 0$.

Check the two formulas for isothermal, reversible work.

$$w = -nRT\ln\frac{V_{final}}{V_{initial}} = -(1.00\ mol)(8.31447\ J{\cdot}K^{-1}{\cdot}mol^{-1})(300\ K)\ln\left(\frac{24.6\ L}{12.3\ L}\right)$$

or $$w = -nRT\ln\frac{P_{initial}}{P_{final}} = -(1.00\ mol)(8.314\,47\ J{\cdot}K^{-1}{\cdot}mol^{-1})(300\ K)\ln\left(\frac{2.00\ atm}{1.00\ atm}\right)$$

$w = -1.73\times 10^3$ J and $q = 1.73\times 10^3$ J

The absolute value of the work done by the system in a reversible expansion is greater than the value obtained in an irreversible expansion. It is possible to show that the maximum amount of work is always done by the system in a reversible expansion.

c) The internal energy of an ideal gas depends only on temperature, and $\Delta U = 0$.

The labels for initial and final states are interchanged. The calculation for reversible work results simply in a change in sign.

$$w = -nRT \ln \frac{V_{\text{final}}}{V_{\text{initial}}} = -(1.00 \text{ mol})(8.314\,47 \text{ J}\cdot\text{K}^{-1}\cdot\text{mol}^{-1})(300 \text{ K}) \ln\left(\frac{12.3 \text{ L}}{24.6 \text{ L}}\right)$$
$$= +1.73 \times 10^3 \text{ J} \quad \text{and} \quad q = -1.73 \times 10^3 \text{ J}$$

The calculation for irreversible work results in a quite different value.

$$w = -P_{ex}\Delta V = -(2.00\ atm)(-12.3\ L) = +24.6\ L \cdot atm = (24.6\ L \cdot atm)(101.325\ J \cdot L^{-1} \cdot atm^{-1})$$
$$= +2.50 \cdot 10^3\ J\ and\ q = -2.50 \cdot 10^3.$$

The value of the work done on the system in a reversible compression is smaller than the value obtained in an irreversible compression. It is possible to show that the minimum amount of work is always done on the system in a reversible compression.