Answer on Question #52499, Physics, Mechanics | Kinematics | Dynamics

A spring with a spring constant k= 2 n/m is used to project a mass around a loop of radius r=2m. To what minimum distance X_{min} must the spring be compressed for the object to go around the loop without falling off?

Solution:

The total mechanical energy in any isolated system of objects remains constant, thus potential energy of spring is equal to kinetic energy of mass and potential energy of mass.

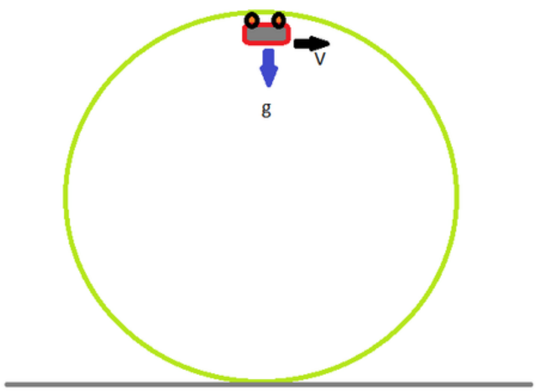
$$U_i = K_f + U_f$$

$$\frac{kx^2}{2} = \frac{mv_f^2}{2} + mgh$$

where k = spring constant, x = amount of compression.

While body is at the top of the loop, Gravitational force acts downwards which is equal to the weight of the car plus weight of the driver. Since we do not need to calculate precise values here, consider the weight of the body only. Centripetal force is a force which keeps a body moving with a uniform speed along a circular path. The sum of all the forces acting on the body will have to equal to the centripetal force.

In order to stay on the loop, body needs to attain the speed such that centripetal force equals to the weight of the body.



Centripetal force

$$F_c = \frac{mv_f^2}{r}$$

Weight

$$P = mg$$

$$\frac{mv_f^2}{r} = mg$$

Thus, the final kinetic energy

$$\frac{mv_f^2}{2} = \frac{mgr}{2}$$

$$U_f = mgh = mg(2r)$$

Hence,

$$\frac{kx^2}{2} = \frac{mgr}{2} + 2mgr$$

$$\frac{kx^2}{2} = \frac{5}{2}mgr$$

$$x_{min} = \sqrt{\frac{5mgr}{k}} = \sqrt{\frac{5mg * 2\text{ m}}{2\text{ N/m}}} = \sqrt{5mg}\text{ m}$$

where g = 9.8 m/s^2 and m = mass.

Answer: $x_{min} = \sqrt{\frac{5mgr}{k}}$