An electric kettle uses 2 resistors, if one is use it takes $t_1 = 6$ mins to boil water, the 2nd resistor takes $t_2 = 8$ mins to boil the same quantity of water. What time it would take if they are connected in

    (i)   series and
    (ii)   parallel

Please explain in detail.

Solution:

Let the first resistor be connected to the source of voltage V, the power of heat then is given by

$$P_1 = \frac{V^2}{R_1},$$

where R_1 – is its resistance.

If the second resistor is connected to the same source, the power of heat is

$$P_2 = \frac{V^2}{R_2},$$

where R_2 – is its resistance.

The energy needed to boil the kettle is the same in both cases, therefore,

$$P_1 t_1 = P_2 t_2$$

$$\frac{V^2}{R_1} t_1 = \frac{V^2}{R_2} t_2$$

$$\frac{R_1}{R_2} = \frac{t_1}{t_2}$$

Since $\frac{t_1}{t_2} = \frac{6 \text{ min}}{8 \text{ min}} = \frac{3}{4}$, we obtain

$$R_1 = \frac{3}{4} R_2$$

If these two resistors are connected in series, their total resistance is given by

$$R_s = R_1 + R_2 = \frac{3}{4} R_2 + R_2 = \frac{7}{4} R_2$$

The power of heat released in this case is given by (connected to the same source of voltage V)

$$P_s = \frac{V^2}{R_s} = \frac{4}{7} \frac{V^2}{R_2} = \frac{4}{7} P_2$$

The energy needed to boil the kettle is given by

$$A = P_s t_s,$$

where t_s – is the time for connection in series to do this. The same energy can be expressed as

$$A = P_2 t_2$$

Therefore,

$$P_s t_s = P_2 t_2$$

$$\frac{4}{7} P_2 t_s = P_2 t_2$$

$$t_s = \frac{7}{4} t_2 = \frac{7}{4} 8 \text{ min} = 14 \text{ min}$$

In the case of parallel connection the total resistance is given by

$$R_p = \frac{R_1 R_2}{R_1 + R_2} = \frac{\frac{3}{4} R_2 R_2}{\frac{3}{4} R_2 + R_2} = \frac{3}{7} R_2$$

The power of heat released in this case is

$$P_p = \frac{V^2}{R_p} = \frac{7}{3} \frac{V^2}{R_2} = \frac{7}{3} P_2$$

Since $P_p t_p = P_2 t_2$ (t_p – is time needed for parallel connection to boil the kettle), we obtain

$$\frac{7}{3} P_2 t_p = P_2 t_2$$

$$t_p = \frac{3}{7} t_2 = \frac{3}{7} 8 \text{ min} = \frac{24}{7} \text{ min} = 3.43 \text{ min}$$

<u>Answer:</u>

    (i)   14 min
    (ii)   3.43 min